**Math 142A Homework Assignment 3**
**Due 11:00pm Friday, October 21, 2022**

1. Let $(r_n)$ be an enumeration of the set $\mathbb{Q}$ of all rational numbers. Show there exists a subsequence $(r_{n_k})$ such that $\lim_{k\to\infty} = +\infty$.

2. Prove that $\liminf s_n = -\limsup(-s_n)$ for every sequence $(s_n)$.
   [Hint: See Definition 10.6 and Exercise 5.4]

3. Let $S$ be a bounded set. Prove that there is an increasing sequence $(s_n)$ in $S$ such that $\lim s_n = \sup S$. Explain why if $\sup S$ is in $S$, it suffices to define $s_n = \sup S$ for all $n$.

4. Let $(s_n)$ be an increasing sequence. Show that $(s_n)$ has no dominant terms.

5. Let $(s_n)$ be a strictly decreasing sequence; that is, $s_n > s_{n+1}$ for every $n \in \mathbb{N}$. Show that every term of $(s_n)$ is a dominant term.

6. Show that a monotonically increasing sequence is bounded if it has a bounded subsequence.

7. Suppose the sequence $(s_n)$ is monotonically increasing and that it has a convergent subsequence. Show that $(s_n)$ converges.

8. Let $c > 0$. Consider the quadratic equation $x^2 - x - c = 0$, where $x > 0$. Define the sequence $(x_n)$ recursively, as follows:

$$x_n = \begin{cases} 1 & \text{if } n = 1 \\ \sqrt{c + x_{n-1}} & \text{if } n > 1 \end{cases}$$

   Prove that $(x_n)$ converges monotonically to the solution of the above quadratic equation.