Math 142A Homework Assignment 1
Due 11:00pm Friday, October 7, 2022

1. Show that $\sqrt{4+2\sqrt{3}}-\sqrt{3}$ is a rational number.
2. Find all rational solutions of the equation $x^8-4x^5+13x^3-7x+1=0$. Be sure to explain how you know you found all the rational solutions.
3. (a) Show $|b|\le a$ if and only if $-a\le b\le a$.
   (b) Prove $\big||a|-|b|\big|\le|a-b|$ for all $a,b\in\mathbb{R}$.
4. Let $a,b\in\mathbb{R}$. Show that if $a<b_1$ for every $b_1>b$, then $a\le b$.
5. Prove that if $a>0$, then there exists $n\in\mathbb{N}$ such that $\dfrac{1}{n}<a<n$.
6. Let $a,b\in\mathbb{R}$. Show that if $a\le b+\frac{1}{n}$ for all $n\in\mathbb{N}$, then $a\le b$.
7. Let (t_n) be a bounded sequence; that is, there exists $M\ge 0$ such that $|t_n|\le M$ for all n. Let (s_n) be a sequence such that $\lim s_n=0$. Prove that $\lim(s_nt_n)=0$.
8. Consider three sequences $(a_n),(b_n)$, and (s_n) such that $a_n\le s_n\le b_n$ for all n, and $\lim a_n=\lim b_n=s$. Prove that $\lim s_n=s$.
9. Suppose (s_n) and (t_n) are sequences such that $|s_n|\le t_n$ for all n and $\lim t_n=0$. Prove that $\lim s_n=0$.
10. Let (s_n) be a sequence that converges.
    (a) Show that if $s_n\ge a$ for all but finitely many n, then $\lim s_n\ge a$.
    (b) Show that if $s_n\le b$ for all but finitely many n, then $\lim s_n\le b$.
    (c) Conclude that if all but finitely many s_n belong to $[a,b]$, then $\lim s_n$ belongs to $[a,b]$.