

Math 142A Homework Assignment 5

Due Thursday, February 18, 2021

1. Show that $f : [1, \infty) \rightarrow \mathbb{R}$ given by $f(x) = \sqrt{x}$ satisfies the $\varepsilon - \delta$ criterion on $[1, \infty)$. Conclude that f is uniformly continuous.
2. Show that if $f : (a, b) \rightarrow \mathbb{R}$ is uniformly continuous, then f is bounded; that is, $f((a, b))$ is bounded.
3. Suppose a continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$ is periodic; that is, there is a number $p > 0$ such that $f(x + p) = f(x)$ for all x . Show that $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous.
4. Exhibit an example of
 - (a) a continuous function $f : (0, 1) \rightarrow \mathbb{R}$ that is not bounded.
 - (b) a bounded continuous function $g : (0, 1) \rightarrow \mathbb{R}$ that is not uniformly continuous.
5. Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a monotone function. Prove that $\lim_{x \rightarrow \infty} f(x) = L$ for some number L if and only if $f([0, \infty))$ is bounded.
6. Show that if $f : [a, b] \rightarrow \mathbb{R}$ is a monotone function satisfying the intermediate value property, then f is continuous.