Math 142A Homework Assignment 1
Due Thursday, January 14, 2021

1. Let S and T be nonempty subsets of $\mathbb{R}$ with the following property:

$$s \leq t \text{ for all } s \in S \text{ and } t \in T.$$

   (a) Show tht S is bounded above and that T is bounded below.

   (b) Prove that $\sup S \leq \inf T$.

   (c) Exhibit an example of such sets S and T where $S \cap T$ is nonempty.

   (d) Exhibit an example of sets S and T where $\sup S = \inf T$ and $S \cap T$ is the empty set.

2. Prove that $\displaystyle\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$ for every natural number n.

3. Suppose that S is a nonempty set of integers that is bounded below. Show that S has a minimum.

   Note: This exercise requires the following definition (which your textbook assumes without formally stating):

   Definition. Let S be a nonempty subset of $\mathbb{R}$.

   (a) If S contains a largest element s_0 [that is, $s_0 \in S$ and $s \leq s_0$ for all $s \in S$], then we call s_0 the *maximum* of S and write $s_0 = \max S$.

   (b) If S contains a smallest element, then we call the smallest element the *minimum* of S and write $s_0 = \min S$.

4. Given a real number a, define $S := \{x \in \mathbb{Q} \mid x < a\}$. Prove that $\sup S = a$.

5. Suppose $a \leq \frac{1}{n}$ for every natural number n. Show that $a \leq 0$.

6. Let x and y be real numbers. Use the triangle inequality to prove that $\big|\, |x| - |y| \,\big| \leq |x + y|$.