Math 142B **Midterm Exam 2 v. A** **Name:** ____________

August 30, 2018 **(Total Points: *25*)** **PID:** ____________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Use the Lagrange Remainder Theorem to show that

$$0 < x - \log(1+x) < \frac{1}{2}x^2 \quad \text{for all } x > 0.$$

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(6 points) 2. Use the Lagrange Remainder Theorem to show that, for every pair of real numbers x and h,

$$\left|\cos(x+h) - [\cos(x) - h\sin(x)]\right| \leq \frac{h^2}{2}.$$

(6 points) 3. Exhibit an example of a function f with each of the given properties. Be sure to briefly explain how you know your example has the required property.

(a) A bounded function $f : [0, 1] \to \mathbb{R}$ that is not integrable.

(b) An unbounded function $f : \mathbb{R} \to \mathbb{R}$ for which its n^{th} Taylor polynomial at $x = 0$ satisfies $\lim_{n\to\infty} p_n(x) = f(x)$ for every x.

(c) A bounded and infinitely differentiable function $f : \mathbb{R} \to \mathbb{R}$ which does not have a Taylor series expansion at $x = 0$.

(6 points) 4. The hyperbolic sine function $\sinh(x)$ and hyperbolic cosine function $\cosh(x)$ are defined for all x and satisfy the following properties:

$$\begin{aligned} \sinh(0) &= 0 \text{ and } \cosh(0) = 1 \\ \sinh'(x) &= \cosh(x) \text{ for all } x \\ \cosh'(x) &= \sinh(x) \text{ for all } x \end{aligned}$$

(a) Verify that

$$p_{2n+1}(x) = \sum_{k=0}^{n} \frac{1}{(2k+1)!} x^{2k+1} = x + \frac{1}{6}x^3 + \cdots + \frac{1}{(2n+1)!} x^{2n+1}$$

is the $(2n+1)^{\text{st}}$ Taylor polynomial for $f(x) = \sinh(x)$ centered at $x_0 = 0$.
(Hint: Verify that $\sinh^{(2k)}(0) = 0$ and $\sinh^{(2k+1)}(0) = 1$ for each nonnegative integer k.)

(b) The $(2n+1)^{\text{st}}$ Cauchy integral remainder for $f(x)$ is

$$R_{2n+1}(x) = \frac{1}{(2n+1)!} \int_0^x f^{(2n+2)}(t)(x-t)^{2n+1}\, dt.$$

Let $M > 0$. Show that $\lim\limits_{n\to\infty} R_{2n+1}(x) = 0$ for all x for which $|x| < M$.