Practice Midterm I

Follow these instructions carefully.

1. No calculators or other electronic computational aids may be used during the exam.
2. You may have one page of notes, but no books or other assistance.
3. Write your name, PID, and section on the cover of your bluebook.
4. Show all your work in the bluebook.
5. No credit will be given for unsupported answers.
6. Present your answer clearly.
   a) Carefully indicate the number of each question and question part.
   b) Try to present your answers in the same order they appear in the exam.
   c) Start each question on a new side of a page.

There are four questions. Each question is worth 10 points.

Question 1.

1. Compute the sum of the binary numbers 1010111 and 1011101. Leave your answer in binary.
2. Convert the binary number 111 to base 10.
3. Is the binary number 1100000011 odd or even?
4. Multiply the binary number 111 by 8. Leave your answer in binary.

Question 2.
Prove the following claim in propositional logic by giving a two column proof:

$$(p \to (q \vee r)) \wedge ((r \wedge p) \to q) \implies p \to q$$

In your proof, you may use the rules described in Figure 1.

Question 3.
Let $P(x)$ be the predicate "x is a bleep." and $Q(x)$ be the predicate "x is a blop." Write a paragraph explaining why $(\exists x)(P(x) \to Q(x))$ is *not* the translation of the sentence "there is a bleep that is a blop," and also why $(\forall x)(P(x) \to Q(x))$ is the translation of the sentence "all bleeps are blops."

Question 4.
Give a direct proof of the following claim: the product of any two odd numbers is an odd number. Be sure that your proof refers to the definition of an odd number.

Equivalence	Name
$\neg\neg p \iff p$	double negation
$p \to q \iff (\neg p) \vee q$	implication
$\neg(p \wedge q) \iff (\neg p) \vee (\neg q)$ $\neg(p \vee q) \iff (\neg p) \wedge (\neg q)$	De Morgan's laws
$p \wedge q \iff q \wedge p$ $p \vee q \iff q \vee p$	commutivity
$p \wedge (q \wedge r) \iff (p \wedge q) \wedge r$ $p \vee (q \vee r) \iff (p \vee q) \vee r$	associativity
$p \wedge p \iff p$ $p \vee p \iff p$	idempotents
$p \to q \iff \neg q \to \neg p$	contraposition

Inference	Name
$\left.\begin{matrix} p \\ q \end{matrix}\right\} \implies p \wedge q$	conjunction
$\left.\begin{matrix} p \\ p \to q \end{matrix}\right\} \implies q$	*modus ponens*
$\left.\begin{matrix} \neg q \\ p \to q \end{matrix}\right\} \implies \neg p$	*modus tollens*
$p \wedge q \implies p$	simplification
$p \implies p \vee q$	addition
$\left.\begin{matrix} p \to q \\ q \to r \end{matrix}\right\} \implies p \to r$	transitivity

Figure 1: Equivalence rules (left) and inference rules (right). For use in Question 2.