Moduli space of Riemann surfaces and conformal field theory

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In his axiomatic approach of conformal field theory (CFT) Graeme Segal introduces the category of Riemann surfaces. Its objects are disjoint circles (closed strings) and its morphisms are Riemann surfaces, thought of as cobordisms. Thus CFT is in particular a study of Riemann's moduli spaces.

The main result is that Segal's category gives rise to an infinite loop space $\mathcal{M}_\infty^+$ [Inventiones, 1997]. An infinite loop space is the analogue of an abelian group in the homotopy category and can be studied with the powerful tools of stable homotopy theory. In an application, infinite families of torsion classes in the homology of moduli spaces are found [with Ib Madsen, Inventiones]. We are also led to a refinement of the Mumford conjecture. The question whether the rational cohomology of $\mathcal{M}_\infty^+$ is a polynomial algebra on even generators is replaced by the question whether a natural map from $\mathcal{M}_\infty^+$ to $Q(\mathbf{C}P_{-1}^\infty)$ is a homotopy equivalence. The latter is a well understood infinite loop space associated to complex projective space.