

String Topology

By Dennis P. Sullivan

There is a diffeomorphism invariant structure in the free loop space of a manifold defined (with Moira Chas) by considering transversal intersections in families of collections of closed curves. At the first stage there is at the level of chains a mapping from the moduli space of surfaces with parametrized input and output boundary components to operators between the function space of mappings of input and output boundaries into the manifold. This structure resembles a combinatorial conformal field theory.

At the second stage the reparametrization symmetry is used to induce an equivariant version of the first stage which suppresses tiny loops. This structure comprises a diffeomorphism invariant string theory associated to each smooth d -manifold. It consists of chain operations parametrized by chains on partial completions of the moduli spaces of punctured Riemann surfaces. One corollary is a natural Lie bialgebra structure on appropriate equivariant chains of the free loop space. If we specialize to dimension two and pass to homology this is the Lie bialgebra discovered in stages by Goldman and Turaev.