THE TOPOLOGY OF SPACES OF EMBEDDINGS OF THE CIRCLE

DEV PRAKASH SINHA

We study the topology of spaces of embeddings of the circle, $\mathrm{Emb}(S^1, M)$, where M is a smooth manifold. We present spectral sequences for both cohomology and homotopy groups of $\mathrm{Emb}(S^1, M)$. When M is Euclidean space, the cohomology spectral sequence agrees through the E_2-term with Vassiliev's spectral sequence, while the homotopy spectral sequence is new, highlighting an advantage of our techniques (reminiscent of advantages of the Postnikov tower over cellular decomposition of a space). We show that these spectral sequences converge when M has dimension greater than or equal to four, as claimed for the cohomology spectral sequence by Kontsevich. Combining our results with recent work of Cattaneo et. al. on generalized Bott-Taubes integrals, we find that the cohomology spectral sequence collapses rationally, giving a complete computation of the rational cohomology of these spaces.

When the dimension of M is three, the questions of convergence and collapse are open for our models, but one can still pull back zero-dimensional cohomology classes (that is, knot invariants). The resulting knot invariants are of finite type and because the combinatorics of chord diagrams (that is, graph cohomology) arises naturally in our spectral sequence, we conjecture that all finite-type invariants arise in this way. The combinatorics of the homotopy spectral sequence, involving free Lie algebras, should open up new questions in quantum algebra. We present preliminary computations with rational coefficients.

We rely heavily on work of Goodwillie and Klein on spaces of embeddings in general. Following a program of Goodwillie's, we find what are essentially cosimplicial models of these spaces, so that our spectral sequences are simply the cohomology and homotopy spectral sequences of a cosimplicial space. In this way, we give topological underpinnings of the appearance of chord diagrams when studying the cohomology of spaces of embeddings of S^1, as when Vassiliev founded the study of finite-type knot invariants, and the appearance of trivalent diagrams when studying the homotopy groups of these spaces.

Department of Mathematics, Brown University, Providence, RI 02906
E-mail address: dps@math.brown.edu