DIRAC OPERATORS ON HOMOGENEOUS LOOP SPACES

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The elliptic genus of a manifold M can be expressed as a power series, where each coefficient is the index of the Dirac operator twisted by a particular bundle. Recognizing these bundled-valued power series from string theory, Witten proposed that the elliptic genus is the index of a formal Dirac operator on the free loop space LM. In his paper "S^1 Actions and Elliptic Genera", Taubes formalized Witten's heuristic arguments, using localization to compute the index by constructing a local Dirac operator in a neighborhood of the constant loops.

If M is a Lie group G or homogeneous space G/H, it is possible to explicitly construct a global Dirac operator on the loop space LM. In this case, the otherwise intractable infinite dimensional analysis is replaced by the representation theory of loop groups, and the universal Dirac operator may be viewed formally as an element of a non-commutative Weil algebra. In order to compute the index of this Dirac operator, we must construct a suitable domain for it to act upon; indeed, by a generalization of a theorem of Bott, the index is completely determined by this choice of domain and range. We model our Hilbert space of functions on LG on the Peter-Weyl decomposition of $L^2(G)$ in terms of the irreducible representations of G. Furthermore, to isolate the LH-equivariant spinors, we use a variant of the BRST construction from string theory, introducing "ghost fields" and then taking their Lie algebra cohomology. Finally, we show that as the central charge tends to infinity, our global Dirac operator reduces to Taubes' local operator, thereby recovering the elliptic genus.