Understanding the Kauffman Bracket Skein Module

The lecture will develop the Kauffman bracket skein algebra of a surface as a tool for interpolating between representation theoretic data and quantum invariants of three manifolds.

The Kauffman bracket skein relation is analogous to the Cayley-Hamilton identity for SL_2. Consequently, the Kauffman bracket skein algebra of a cylinder over a surface is a quantization of the $SU(2)$ characters of the fundamental group of the surface with respect to the Poisson bracket coming from the standard symplectic structure on the $SU(2)$ characters. Furthermore the algebra of observables in lattice gauge field theory based on quantum SL_2 is the Kauffman bracket skein module. There is a direct construction of topological quantum field theory based on the Kauffman bracket skein module.

Next, the relationship between the Kauffman bracket skein algebra of a torus and the noncommutative torus will be explained. This leads to a noncommutative version of the A-polynomial of knots. The noncommutative A-polynomial interpolates between representation theoretic data about knot complements and the Jones polynomial.

Finally, the Yang-Mills measure in the Kauffman bracket skein module is a locally defined, diffeomorphism invariant trace, that quantizes integration against the symplectic measure on the space of $SU(2)$ characters of the fundamental group of a surface. The measure yields a visual calculus for evaluation of such integrals. It is also the path integral that underlies quantum invariants of three manifolds.

References

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