PATH INTEGRALS ON MANIFOLDS

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ABSTRACT. A typical path integral is a **informal** expression of the form

$$\frac{1}{Z}\int_{\mathcal{F}} f(x)e^{-E(x)}\mathcal{D}x \tag{1}$$

where $\mathcal{F}$ is a space of maps from one manifold to another, f is a real valued function on $\mathcal{F}$, $E(x)$ is the energy of the map x, $\mathcal{D}x$ is "Lebesgue measure" and Z is a normalization constant. The use of path integrals for "quantizing" classical mechanical systems (whose classical energy is E) started with Feynman in [2] with very early beginnings being traced back to Dirac [1]. Path integrals are still heavily used by physicists for both the quantum mechanics of elementary particles and more recently for conjecturing new topological invariants of manifolds. In this talk, I will discuss joint work with Lars Andersson, on defining the path integral in Eq. (1) when $\mathcal{F}$ is the space of continuous maps (x) from $[0,T]$ to a compact Riemannian manifold (M) and $E(x)$ is the standard Riemannian energy of the path x. The idea is to approximate $\mathcal{F}$ by finite dimensional subspaces consisting of broken geodesics and then to pass to the limit of finer and finer approximations. This method of defining (1) leads to a quantum mechanical system whose Hamiltonian is of the form $H = -\frac{1}{2}\Delta + \kappa\text{Scal}$, where Δ is the Laplacian on M, Scal is the scalar curvature of M and κ is a constant which depends on how one interprets $\mathcal{D}x$.

REFERENCES

[1] P. A. M. Dirac, Physikalische Zeitschrift der Sowjetunion **3** (1933), 64.
[2] R. P. Feynman, *Space-time approach to non-relativistic quantum mechanics*, Rev. Modern Physics **20** (1948), 367–387.

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