Section 5.2 Problem 12

We first calculate the coefficients c_n of the exponential series for e^x. They are given by the following formula (observe that in the previously posted solution the factor in front of the integral was wrong):

$$c_n = \frac{1}{2\ell}\int_{-\ell}^{\ell} e^x e^{-i\frac{n\pi}{\ell}x}\, dx \ =$$

$$= \frac{1}{2\ell(1-i\frac{n\pi}{\ell})}(e^{\ell-in\pi} - e^{in\pi-\ell}) \ =$$

$$= \frac{(-1)^n(\ell+in\pi)}{\ell^2+n^2\pi^2}\sinh(\ell),$$

since $e^{in\pi} = (-1)^n$ and $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$. Similarly, we obtain for the coefficients $\tilde{c}_n$ of the exponential series for e^{-x} that

$$\tilde{c}_n \ = \ \frac{(-1)^n(\ell-in\pi)}{\ell^2+n^2\pi^2}.$$

As $\cosh x = \frac{1}{2}(e^x + e^{-x})$, we obtain

$$\cosh x = \sum_n \frac{(-1)^n\ell}{\ell^2+n^2\pi^2}\ e^{n\pi ix/\ell}.$$