MATH 104 MIDTERM WINTER 2000

Instructor: Wenzl

Justify your answers!

1. (a) Prove that $(4k+2, 5k+3)$ is either 1 or 2.
   (b) Determine all k for which the *gcd* is equal to 2.
2. Compute all solutions of $x^2 + x + 1 \equiv 0 \mod 49$. (No credit if you try all numbers from 1 to 49).
3. (a) Find all solutions of the following Diophantine equation: $65x - 43y = 3$.
   (b) Prove or disprove: $\{0, 65 \cdot 1, 65 \cdot 2, \ldots 65 \cdot 42\}$ is a complete residue system mod 43.

MATH 104 MIDTERM WINTER 97

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Justify your answers!

1. (a) Compute $18 \cdot 19 \cdot 20 \ \ldots \ 31 \cdot 32 \cdot 33 \mod 17$.
   (b) Compute $4^{44} \mod 23$.
2. (a) Compute the smallest positive integer n such that $n \equiv 5 \mod 17$ and $n \equiv 2 \mod 8$.
   (b) Which is the second smallest positive number which satisfies the congruence in (a)?
3. Show that $\sqrt{7}$ is irrational, using uniqueness of prime factorization (partial credit for other methods).
4. Let n be an **odd** number such that $5 \not| n$. Show that $n^4 + 4^n$ is not a prime for $n > 1$.