Math 102
Winter '08
Homework 1.

1.2

$\boxed{\#3}$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 & 2 \end{array}\right) \overset{R2-R3}{\sim} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 \end{array}\right)$$

$$\Rightarrow \begin{array}{l} v+z=4 \\ z=2 \\ u+w=2 \end{array} \quad \Rightarrow \quad \begin{array}{l} v=2 \\ z=2 \\ w=2-u \end{array}$$

Soln in parametric form:

$$\begin{pmatrix} u \\ v \\ w \\ z \end{pmatrix} = \begin{pmatrix} u \\ 2 \\ 2-u \\ 2 \end{pmatrix} = u\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 2 \\ 2 \end{pmatrix} \quad \text{is a } \boxed{\text{line}}$$

If 4^{th} plane $u=-1$ is included, sol'n becomes:

$$\begin{array}{l} v=2 \\ z=2 \\ u=-1 \\ w=2-(-1)=3. \end{array}$$

and so the intersection is the $\boxed{\text{point}}$ $(-1,2,3,2)$

$\boxed{\#10}$ $(0,y_1)$, $(1,y_2)$, $(2,y_3)$ lie on a straight line when

$y-y_1=(y_2-y_1)(x-0) \iff y-y_1=(y_2-y_1)x$

so, $y_3-y_1=(y_2-y_1)2 \Rightarrow y_3=2y_2-y_1$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ 2y_2-y_1 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \quad \text{where } s,t \in \mathbb{R}.$$

1.3

$\boxed{\#3}$ $2x - 4y = 6$ (2)

$-x + 5y = 0$ (3)

Subtract $-1/2$ times (2) from (3):

$$-\frac{1}{2}(2x - 4y = 6) \quad -x + 5y = 0 \rightarrow \begin{array}{l} -x + 2y = -3 \\ -x + 5y = 0 \end{array} \xrightarrow{(3) \to (3)-(2)} \begin{array}{r} -x + 2y = -3 \\ 3y = 3 \end{array}$$

$\Rightarrow \boxed{y = 1}$, $-x + 2 = -3 \Rightarrow \boxed{x = 5}$

If the RHS changes to $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$, soln becomes $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$.

$\boxed{\#18}$

(A) If (x, y, z) & (X, Y, Z) are 2 solutions, then another solution is $\frac{1}{2}(x, y, z) + \frac{1}{2}(X, Y, Z)$.

(B) If 25 planes meet at 2 points they also meet at the entire line containing those 2 points.

$\boxed{\#31}$ Gaussian Elim fails for

$a = 0$ (zero column)

$a = 2$ (1^{st} & 2^{nd} cols equal)

$a = 4$ (2^{nd} & 3^{rd} rows equal).

1.4

#6

$$a_{ij} = i + j \Rightarrow A = \begin{pmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$

$$b_{ij} = (-1)^{i+j} \Rightarrow B = \begin{pmatrix} -1^{1+1} & -1^{1+2} \\ -1^{2+1} & -1^{2+2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}.$$

#10

(A) $\boxed{\text{True}}$ Let $B = (b_1 | b_2 | b_3 | \cdots | b_n)$, where $b_1 = b_3$.

Then $AB = A(b_1 | b_2 | b_3 | \cdots | b_n) = (Ab_1 | Ab_2 | Ab_3 | \cdots | Ab_n)$

But $b_1 = b_3 \Rightarrow Ab_1 = Ab_3$

$\therefore$ Columns 1 & 3 of AB are the same.

(B) $\boxed{\text{False}}$ $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 2 & 3 \\ 4 & 4 & 2 \\ 3 & 2 & 3 \end{pmatrix}$ rows 1 & 3 same

$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 2 & 3 \\ 4 & 4 & 2 \\ 3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 3 \\ 3 & 2 & 3 \\ 4 & 4 & 2 \end{pmatrix}$ rows 1 & 3 not same.

(C) $\boxed{\text{True}}$ Let $A = \left(\begin{array}{c} a_1 \\ \hline \vdots \\ \hline a_n \end{array}\right)$ with $a_1 = a_3$.

Then $AB = \left(\begin{array}{c} a_1 \\ \hline \vdots \\ \hline a_n \end{array}\right) B = \left(\begin{array}{c} a_1 B \\ \hline \vdots \\ \hline a_n B \end{array}\right)$

and since $a_1 = a_3$, $a_1 B = a_3 B$ so the 1st and 3rd rows of AB are the same.

(D) $\boxed{\text{False}}$ $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -2 & 6 \end{pmatrix}, \text{ so}$$

$$(AB)^2 = \begin{pmatrix} 4 & 3 \\ -2 & 6 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ -2 & 6 \end{pmatrix} = \begin{pmatrix} 10 & -6 \\ -20 & 30 \end{pmatrix}.$$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}; \quad B^2 = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\text{so } A^2B^2 = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 30 & 0 \\ 0 & 12 \end{pmatrix} \neq (AB)^2.$$

$\boxed{\#32}$

(A)
$$\begin{matrix} x = 2y \\ x + y = 39 \end{matrix} \rightarrow \begin{matrix} x - 2y = 0 \\ x + y = 39 \end{matrix} \rightarrow \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 39 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 1 & 39 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 3 & 39 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 1 & 13 \end{array}\right)$$

$$\Rightarrow \boxed{y = 13}, \quad x - 2(13) = 0 \Rightarrow \boxed{x = 26}$$

(B)
$$\begin{matrix} 5 = 2m + c \\ 7 = 3m + c \end{matrix} \rightarrow \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & 1 & 7 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & 1 & 5 \\ 0 & 1-\frac{3}{2} & 7-\frac{3}{2}(5) \end{array}\right) = \left(\begin{array}{cc|c} 2 & 1 & 5 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right)$$

$3 - \frac{3}{2}(2) = 0$

$$\Rightarrow \boxed{c = 1}, \quad 2m + 1 = 5 \Rightarrow \boxed{m = 2}$$