We consider the initial value problem associated with the 3-dimensional wave equation

$$u_{tt} = c^2 \Delta u + f(\mathbf{x}, t), \qquad u(\mathbf{x}, 0) = 0, \quad u_t(\mathbf{x}, 0) = 0. \tag{$*$}$$

(a) Let s be fixed and define

$$v^{(s)}(\mathbf{x}, t) = \frac{1}{4\pi c^2 t} \int\int_S f(\mathbf{x}, s) dS,$$

where S is the sphere given by $|\xi - \mathbf{x}| = ct$. This is a solution of the wave equation $v_{tt} = c^2 \Delta v$ with which initial conditions?

(b) Show that $u(\mathbf{x}, t) = \int_0^t v^{(s)}(\mathbf{x}, t - s) ds$ is a solution of $(*)$.

(c) Find the solution of $(*)$ with general initial conditions $u(\mathbf{x}, 0) = \phi(\mathbf{x})$, $u_t(\mathbf{x}, 0) = \psi(\mathbf{x})$. *Hint* : You can obtain the solution for (c) by adding a solution of the usual wave equation $v_{tt} = c^2 \Delta v$ with suitable initial conditions to the solution in (b).