PRACTICE PROBLEMS MIDTERM II

1. Assume that $f : \mathbf{R} \to \mathbf{R}$ has three derivatives and define

$$F(x) = f(0) + f'(0)x + \int_0^x (x-t)f''(t)\ dt,$$

for all x.
(a) Show that $F'(x) = f'(x)$ (Hint: You can pull out x from the integral - do NOT use the Cauchy integral remainder theorem)
(b) Use part (a) and the Identity Criterion (see book, page 117) to show that $F(x) = f(x)$ for all x.

2. Let $f : \mathbf{R} \to \mathbf{R}$ be a function which has three derivatives. Assume that $f'(0) = 0$ and $f''(0) = 1$. Prove that $x_0 = 0$ is a local minimizer, i.e. there exists a $\delta > 0$ such that $f(x) > f(0)$ if $|x - 0| < \delta$.

3. Assume that $f : \mathbf{R} \to \mathbf{R}$ has derivatives of all orders such that

$$|f^{(k)}(x)| \leq k! \quad \text{for all } x \in (-1,1),\ k \in \mathbf{N}.$$

Prove that $f(x)$ is given by its Taylor series for all $x \in (-1,1)$, i.e.

$$f(x) \ = \ \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}\ x^k \quad \text{for all } x \in (-1,1).$$

4. Do the following problems: Section 8.2: 1, Section 8.3: 1. Review homework problems.