# Math 142B : Practice Midterm 1

**1. True/False:** Circle the correct answer. No justifications are needed in this exercise.

(1) The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^{3n}}{\sqrt{n}}$ is $(-1,1)$. **T / F**

(2) Let $(f_n)$ be a sequence of continuous functions on $[0,1]$. Assume that $f_n \to f$ uniformly on $[0,1]$, for some function $f:[0,1]\to\mathbb{R}$. Then $f$ is bounded on $[0,1]$. **T / F**

(3) Let $(f_n)$ be a sequence of continuous functions on $[0,1]$. Assume that $f_n \to f$ pointwise on $[0,1]$, for some function $f:[0,1]\to\mathbb{R}$. Then $f$ is continuous on $[0,1]$. **T / F**

(4) Suppose that $(a_k)$ is a sequence of real numbers such that the series $\sum_{k=0}^{\infty} 2^k a_k$ converges. Then the power series $\sum_{k=0}^{\infty} a_k x^k$ converges uniformly on $[0,1]$. **T / F**

(5) Let $(a_k)$ be a sequence of real numbers such that the series $\sum_{k=1}^{\infty} a_{2k}$ and $\sum_{k=1}^{\infty} a_{2k-1}$ are convergent. Then the power series $\sum_{k=1}^{\infty} a_k x^k$ converges uniformly on $[-1,1]$. **T / F**

**2.** (a) State the formula for the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$.

For (b)-(c), consider functions $f_n : S \to \mathbb{R}$, for every $n \in \mathbb{N}$, and $f : S \to \mathbb{R}$.

(b) State what it means for the sequence $(f_n)$ to converge pointwise on $S$ to $f$.

(c) State what it means for the sequence $(f_n)$ to converge uniformly on $S$ to $f$.

(c) State what it means for the sequence $(f_n)$ to be uniformly Cauchy on $S$.

For (d)-(e), consider functions $g_k : S \to \mathbb{R}$, for every $k \in \mathbb{N}$.

(d) State what it means for the series of functions $\sum_{k=0}^{\infty} g_k$ to satisfy the Cauchy criterion uniformly on $S$.

(e) State the Weierstrass M-test for the series of functions $\sum_{k=0}^{\infty} g_k$ on the set $S$.

**3.** Let $f_n : (0,\infty) \to \mathbb{R}$ given by $f_n(x) = \frac{1}{nx+1}$ and $f : (0,\infty)\to\mathbb{R}$ given by $f(x) = 0$.

(a) Prove that $f_n \to f$ pointwise on $(0,\infty)$.

(b) Prove that $f_n \to f$ uniformly on $(\delta,\infty)$, for every $\delta > 0$.

(c) Prove that the sequence $(f_n)$ does not converge uniformly to $f$ on $(0,\infty)$.

**4.** (a) Let $(f_n)$ be a sequence of functions on a set $S$, and suppose that $f_n \to f$ uniformly on $S$, for some bounded function $f : S \to \mathbb{R}$. Prove that $f_n^2 \to f^2$ uniformly on $S$.

(b) Prove that the series $\sum_{n=1}^{\infty} \frac{x^{n^2}}{2^n}$ has radius of convergence 1 and converges uniformly to continuous function on $[-1,1]$.

**5.** (a) Prove that the series $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$ converges uniformly on $[-M,M]$, for every $M > 0$.

(b) Prove that the series $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$ does not converge uniformly on $\mathbb{R}$.

(c) Define $f : \mathbb{R} \to \mathbb{R}$ by letting $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$.

Prove that $f$ is differentiable on $\mathbb{R}$ and $f'(x) = xf(x)$, for every $x \in \mathbb{R}$.