LIE GROUPS: 2nd Assignment

As before, whenever I refer to the book by Brian Hall, I am referring to the first edition.

1. (a) Problem 27 on page 62.
   (b) Show that $U(n)$ has a universal cover which is homeomorphic to $\mathbf{R} \times SU(n)$. (You may assume that $SU(n)$ is simply connected). Calculate the fundamental group of $U(n)$.
2. (a) Problem 10 on page 89.
   (b) Recall that we defined the universal cover $\hat{G}$ of an Lie group as the set

$$\{\gamma : [0,1] \to G \textit{ continuous}, \ \gamma(0) = I\}/\textit{homotopy},$$

   i.e. homotopic paths get identified. Show that $\hat{G}$ is a group, with multiplication of paths defined as in part (a). (Ask me if this is too vague).
3. (a) Problem 13 on page 90
   (b) Show that any representation of $\hat{Sl}(n, \mathbf{R})$ is well-defined on the quotient

$$\hat{Sl}(n, \mathbf{C})/\pi_1(Sl(n, \mathbf{C})) \cong Sl(n, \mathbf{C}).$$

   *Hint*: You may use that $\hat{Sl}(n, \mathbf{R})$ has a Lie algebra which is isomorphic to $sl(n, \mathbf{R})$ such that any homomorphism $\Phi : \hat{Sl}(n, \mathbf{R}) \to Gl(m, \mathbf{C})$ induces a Lie algebra homorphism $\psi : sl(n, \mathbf{R}) \to gl(m, \mathbf{C})$.
   (c) Show that $\hat{Sl}(n, \mathbf{R})$ can not be a matrix Lie group.