MATH 21D FINAL SPRING 2002 **Name:** **Section :**

Justify your answers! Put all the essential steps of your solution on this sheet!

1. Compute the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n}{n^3}(x-3)^n$.

2. Determine whether the following series converge.
(a) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+n}}$ (b) $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$

3. Find the general solution of $y' + \frac{1}{t}y = \cos t$.

4. Solve the initial value problem $y'' + 2y' + 2y = 0$, $y(0) = 1$ and $y'(0) = 2$.

5. Determine the general solution of $y'' - 4y' - y = e^{3t}$.

6. (a) Compute the Laplace transform of the function $f(t)$ defined by $f(t) = t - u_1(t)(t-1)$ and $u_1(t) = 0$ for $t < 1$ and $u_1(t) = 1$ for $t \geq 1$. (In notation used in class, $f(t) = t - sh_1(t)$).
(b) Compute the inverse Laplace transform of $\frac{s}{s^2+2s+5} + \frac{1}{s^2-3s+2}$.

7. Solve the following initial value problem via Laplace transformation: $y'' + 4y = g(t)$, where $g(t) = 1$ for $0 \leq t < 1$ and $g(t) = 0$ for $t \geq 1$, and where $y(0) = 0 = y'(0)$.

8. Find a particular solution of the differential equation $ty'' - (1+t)y' + y = t^2 e^{2t}$ via variation of parameters. You may use that the homogeneous differential equation has solutions $y_1(t) = 1+t$ and $y_2(t) = e^t$.

9. (a) Find the recursion relation for the coefficients a_n of a power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ of the differential equation $y'' - xy' + 2y = 0$
(b) Determine all solutions in (a) for which $a_1 = 0$.

10. (a) Show that $x = 0$ is a regular singular point for the differential equation $3x^2y'' + 2xy' + x^2y = 0$ and compute the roots of its indicial equation.
(b) Compute the recursion relation for the coefficients of the power series solution $y(x) = x^r \sum_{n=0}^{\infty} a_n x^n$ of (a), for r the larger root of the indicial equation.
(c) Compute the coefficients a_1, a_2 and a_3 if $a_0 = 1$.