MATH 21C FINAL FALL 2002 **Name:** **Section :**

Justify your answers! Put all the essential steps of your solution on this sheet!

1. Compute the equation of the tangent plane for the surface $x^2y + yz = 6$ at the point (1,2,2).

2. Is there a plane which contains the four points (1,1,0), (1,0,1), (0,0,2) and (2,2,0)?

3. (a) In descending a hill whose equation is given by $z = f(x, y) = 99 - x^2 + 2x - 1.5y^2$, in which direction should a person standing at the point (3,1,94.5) initially proceed so as to descend as quickly as possible?
(b) Is there any direction $\mathbf{u}$ for which the directional derivative $D_{\mathbf{u}}f(3, 1)$ is larger than 6? Why or why not?

4. Compute the double integral $\int\int_D x dA$, where D is the triangle with corners $(0, 0)$, $(1, 1)$ and $(1, 0)$.

5. Compute the integral $\int\int_D e^{x^2+y^2} dA$, where D consists of all points (x, y) with $x^2 + y^2 \leq 9$ and $x \geq 0$.

6. Compute the surface area of the hyperbolic paraboloid $z = x^2 - y^2$ which lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

7. Find the maximum und minimum of the function $f(x, y, z) = 2x - z$ on the ellipsoid given by $x^2 + 10y^2 + z^2 = 5$, using Lagrange multipliers.

8. Compute max/min and saddle points, if any, for the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

9. (a) Describe the part of the ball of radius 2 and center at the origin which lies below the cone $z = -\sqrt{x^2 + y^2}$ in spherical coordinates.
(b) Let $f(x, y, z) = x^3 + \sin yz$ and let ρ, θ, ϕ be spherical coordinates. Find an expression of $\partial f/\partial\phi$. (It may contain ordinary as well as spherical coordinates).

10. Set up the triple integral which computes the volume of the solid between the plane $z-2x-y = 4$ and the paraboloid $z = x^2 + y^2 + y$. You need not compute the integral.

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1. Compute the equation of the tangent plane for the surface $x^2y + yz = 6$ at the point (1,2,2).

2. Compute the integral $\int\int_D \sin(x^2+y^2)dA$, where D consists of all points (x, y) with $x^2+y^2 \leq 9$ and $x \geq 0$.

3. Compute the double integral $\int\int_D x dA$, where D is the triangle with corners $(0, 0)$, $(1, 1)$ and

$(0, 1)$.

4. (a) In descending a hill whose equation is given by $z = f(x, y) = 99 - x^2 + 2x - 1.5y^2$, in which direction should a person standing at the point (3,1,94.5) initially proceed so as to descend as quickly as possible?
(b) Is there any direction $\mathbf{u}$ for which the directional derivative $D_{\mathbf{u}} f(3, 1)$ is larger than 6? Why or why not?

5. Compute the surface area of the hyperbolic paraboloid $z = x^2 - y^2$ which lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

6. Is there a plane which contains the four points (1,0,1), (0,2,2), (2,0,0) and (0,1,1)?

7. Find the maximum und minimum of the function $f(x, y, z) = x - 2z$ on the ellipsoid given by $x^2 + 10y^2 + z^2 = 5$, using Lagrange multipliers.

8. Set up the triple integral which computes the volume of the solid between the plane $z - x - 2y = 4$ and the paraboloid $z = x^2 + y^2 + x$. You need not compute the integral.

9. Compute max/min and saddle points, if any, for the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$.

10. (a) Describe the part of the ball of radius 3 and center at the origin which lies below the cone $z = -\sqrt{x^2 + y^2}$ in spherical coordinates.
(b) Let $f(x, y, z) = y^3 + \cos xz$ and let ρ, θ, ϕ be spherical coordinates. Find an expression of $\partial f / \partial \phi$. (It may contain ordinary as well as spherical coordinates).