MATH 210B WINTER 2001: FINAL

Instructor: Hans Wenzl

1. Solve the integral equation $f(x) = \sin(x) + \lambda \int_0^\pi \cos(x+y) f(y) dy$ (Hint: $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$). Does one get a solution for all values of λ?

2. (a) Compute $\lim_{n\to\infty} \frac{1}{n} \cos nx$ with respect to the L^2 norm on the interval $[-\pi, \pi]$.
(b) Is the differential operator d/dx a continuous operator on the space of differentiable functions on $[-\pi, \pi]$? Either give a counter example or prove continuity of the operator.

3. (a) Show that $x\delta(x) = 0$ (Note: you can not show this by evaluating the left hand side at all points). Do all your integrations over an interval $[a, b]$ with $a < x < b$.
(b) Show that $LG(t, t') = \delta(t - t')$ for $L = d^2/dt^2 + \omega_0^2$ and $G(t, t') = \theta(t - t')\frac{\sin(\omega_0(t-t'))}{\omega_0}$.

4. Let $f(x) = |x|$ for $-\pi \leq x \leq \pi$.
(a) Find the Fourier coefficients (a_k) and (b_k) for f, defined by

$$a_k = \int_{-\pi}^{\pi} f(x)\cos(x)dx, \quad b_k = \int_{-\pi}^{\pi} f(x)\sin(x)dx.$$

Let $f_n = \frac{1}{\pi}[a_0/2 + \sum_{k=1}^n a_k \cos(kx) + b_k \sin(kx)]$.
(b) Does f_n converge to f in L^2 norm?
(c) Does f_n converge to f uniformely?