MATH 20E PRACTICE MIDTERM I

NO BOOKS, NO NOTES!! (except for 1 'cheat sheet')

1. (a) The movement of a particle is described by the path $\mathbf{R}(t) = (5-t, 3-t^2, t)$ for $0 \leq t \leq 1$. Compute its position and direction (velocity vector) at $t = 1$.
   (b) Assume that the particle continues flying in a straight line for $t \geq 1$ in the same direction as for $t = 1$. Where does it hit the yz plane?

2. A cardbord leans against the sphere $x^2 + y^2 + (z-3)^2 = 9$ at the point (2,-2,4). Find the equation of the line which is the intersection of the cardbord with the xy-plane, i.e with $z = 0$.

3. (a) Prove that $\nabla \times (\nabla f) = 0$ for the scalar function $f(x, y, z)$.
   (b) Let $\mathbf{F}(x, y, z) = (2xy, x^2 + ayz, y^2)$, where a is a constant. Using (a), show that $\mathbf{F}$ can not be conservative (i.e. it can not be the gradient of a scalar function $f(x, y, z)$) for all but possibly one value of a. Which value?

4. A bug finds itself in a toxic environment. The toxicity level is given by $T(x, y) = 2x^2 - 4y^2 + x^3$. The bug is at $(-1, 2)$.
   (a) In which direction should it move to lower the toxicity the fastest.
   (b) Find the second order Taylor approximation of $T(x, y)$ at (0,0).