FINAL MATH 20E FALL 03

No calculators, no books, only one cheat sheet allowed. Justify your answers!

1. Compute the line integral $\int_C 3y^2dx + xdy$, where C is the straight line from (0,1) to (1,2).
2. Compute the integral $\int\int_S z + x\ dS$, where S is the part of the plane $2x + y + z = 6$ with $x \geq 0$, $y \geq 0$ and $z \geq 0$.
3. Let $\mathbf{F}_1(x, y, z) = (0, y^2, x)$ and $\mathbf{F}_2(x, y, z) = (z, 2, x)$, and let C be a curve with initial point (0,0,0) and endpoint (1,2,1). For each of the line integrals $\int_C \mathbf{F}_1 \cdot d\mathbf{R}$ and $\int_C \mathbf{F}_2 \cdot d\mathbf{R}$ either compute its value or give a reason why it can not be computed.
4. Compute the volume of the region D given by $1 \leq x^2 + y^2 + z^2 \leq 4$ and $z \geq 0$ via a triple integral.
5. Compute the flux of the vector field $\mathbf{F}(x, y, z) = (x, 1, z^2)$ through the half cylinder S given by $x^2 + y^2 = 4$, $y \geq 0$ and $0 \leq z \leq 1$, with the normal vector pointing away from the origin.
6. Compute $\int\int_S curl\mathbf{F} \cdot d\mathbf{S}$, where S is given by $x^2 + y^2 + (z - 1)^2 = 2$ and $z \geq 0$, and $\mathbf{F}(x, y, z) = (-e^{\sin z}y, x + \sin z, \cos x)$, and the normal vector pointing outside the sphere.
7. Let $\mathbf{F}(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.

   (a) Compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ for C the unit circle parametrized counterclockwise.

   (b) State Green's theorem and explain why the result in (a) does not contradict it.

   (c) Compute $\int_{\tilde{C}} \mathbf{F} \cdot d\mathbf{R}$ for $\tilde{C}$ a curve going around the origin counterclockwise, and containing C in its interior.
8. Let $\mathbf{F}(x, y, z) = (y + z, x - y - z^2, 1 + z)$.
   (a) Compute $\int\int_{S_1} \mathbf{F} \cdot d\mathbf{S}$, where S_1 is the unit disk in the xy plane, i.e. $S_1 = \{(x, y, 0),\ x^2 + y^2 \leq 1\}$, with the normal vector showing upwards.

   (b) Compute $\int\int_{S_2} \mathbf{F} \cdot d\mathbf{S}$, where S_2 is the surface given by $z = 1 - e^{1-x^2-y^2}$ and $z \leq 0$, with the normal vector pointing downwards at $(0, 0, 1 - e)$. ($Hint$: If you try to solve part (b) directly, you will end up with some very ugly integrals. Consider the closed surface consisting of both S_1 and S_2).

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1. Compute the line integral $\int_C y^2 dx + x dy$, where C is the straight line from (1,0) to (2,1).
2. Compute the integral $\int\int_S z + x \, dS$, where S is the part of the plane $x + 2y + z = 6$ with $x \geq 0$, $y \geq 0$ and $z \geq 0$.
3. Compute the volume of the region D given by $1 \leq x^2 + y^2 + z^2 \leq 9$ and $z \geq 0$ via a triple integral.
4. Let $\mathbf{F}_1(x, y, z) = (0, y, x^2)$ and $\mathbf{F}_2(x, y, z) = (z, 2, x)$, and let C be a curve with initial point (0,0,0) and endpoint (2,2,1). For each of the line integrals $\int_C \mathbf{F}_1 \cdot d\mathbf{R}$ and $\int_C \mathbf{F}_2 \cdot d\mathbf{R}$ either compute its value or give a reason why it can not be computed.
5. Compute the flux of the vector field $\mathbf{F}(x, y, z) = (x, 1, z^2)$ through the half cylinder S given by $x^2 + y^2 = 4$, $y \geq 0$ and $0 \leq z \leq 2$, with the normal vector pointing outside.
6. Let $\mathbf{F}(x, y) = (\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2})$.

   (a) Compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ for C the unit circle parametrized counterclockwise.

   (b) State Green's theorem and explain why the result in (a) does not contradict it.

   (c) Compute $\int_{\tilde{C}} \mathbf{F} \cdot d\mathbf{R}$ for $\tilde{C}$ a curve going around the origin counterclockwise, and containing C in its interior.
7. Compute $\int\int_S curl\mathbf{F} \cdot d\mathbf{S}$, where S is given by $x^2 + y^2 + (z - 1)^2 = 2$ and $z \geq 0$, and $\mathbf{F}(x, y, z) = (-e^{z^2} y, x + \sin z, \sin x)$, with the normal vector pointing away from the origin.
8. Let $\mathbf{F}(x, y, z) = (y + z^2, x - y + z, 1 + z)$.
   (a) Compute $\int\int_{S_1} \mathbf{F} \cdot d\mathbf{S}$, where S_1 is the unit disk in the xy plane, i.e. $S_1 = \{(x, y, 0),\ x^2 + y^2 \leq 1\}$, with the normal vector showing upwards.

   (b) Compute $\int\int_{S_2} \mathbf{F} \cdot d\mathbf{S}$, where S_2 is the surface given by $z = 1 - e^{1-x^2-y^2}$ and $z \leq 0$, with the normal vector pointing downwards at $(0, 0, 1 - e)$. ($Hint:$ If you try to solve part (b) directly, you will end up with some very ugly integrals. Consider the closed surface consisting of both S_1 and S_2).