

Justify your answers!

1. Is the vector  $\mathbf{b} = (1, -3, 1)$  in the span of the vectors  $\mathbf{a}_1 = (1, 2, 2)$  and  $\mathbf{a}_2 = (3, 1, 1)$ ?

2. Compute all solutions of  $A\mathbf{x} = \mathbf{b}$ , where  $A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 3 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ .

3. Let  $\mathcal{B} = \{x + 1, x - 1\}$ . Compute the coordinate vector  $[2x + 6]_{\mathcal{B}}$ .

*more problems on backside*

4. (a) Compute the  $LU$ -decomposition  $A = LU$  of  $A = \begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$ .

(b) Show for an arbitrary  $n \times n$  matrix  $A$  that  $\det(A) = u_{11}u_{22} \cdots u_{nn}$ , the product of the diagonal entries of the matrix  $U$  in the  $LU$ -decomposition.

5. Compute a basis for the column space of the matrix  $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 7 & 4 \\ 3 & 9 & 6 & 6 \end{bmatrix}$ . What is its rank?