RECYCLED PROBLEMS FROM OLD EXAMS

1. Let $\mathcal{B} = \{(1,0,0)\ (3,2,1),\ (0,0,2)\}$. You can assume that $\mathcal{B}$ is a basis for $\mathbf{R}^3$.
   (a) Which vector $\mathbf{x}$ has the coordinate vector $[\mathbf{x}]_{\mathcal{B}} = (1,-1,2)$.
   (b) Determine the coordinate vector of $\mathbf{y} = (2,-2,3)$.

2. Compute the dimension of the span of the vectors $(1,4,3)$, $(1,2,1)$ and $(1,1,0)$.

3. Compute the singular value decomposition for the matrix $A = \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix}$

4. Find all eigenvectors with eigenvalue 1 of the matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$.

5. Find all possible values for x and y such that the matrix $A = \begin{bmatrix} 0.6 & x \\ y & 0.6 \end{bmatrix}$ is an *orthogonal* matrix.

6. Find a good approximation for the vector $\begin{bmatrix} .8 & .6 \\ .2 & .4 \end{bmatrix}^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ for n very large (say $n = 100$).

7. Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear map with eigenvectors $(1,3,2)$ (with eigenvalue 1) and $(2,1,4)$ (with eigenvalue 2). What is $L(0,5,0)$?

8. Let S be the row space of the matrix $A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ -1 & -1 & 5 & 1 \end{bmatrix}$.
   (a) Compute the QR-factorization for the matrix A^T (not A itself).
   (b) Compute $P_S(\mathbf{x})$, the orthogonal projection onto S for the vector $\mathbf{x} = (2,2,2,4)$.

9. Let V be an inner product space, and let $\mathbf{v}$ be a vector in V, $\mathbf{v} \neq 0$, and consider the map $P : V \to V$, $P(\mathbf{x}) = \mathbf{x} - \frac{\langle \mathbf{x},\mathbf{v}\rangle}{\langle \mathbf{v},\mathbf{v}\rangle}\mathbf{v}$.
   (a) Compute $P(1,0,1)$ for $\mathbf{v} = (1,1,2)$.
   (b) Show that $P(\mathbf{x})$ is in the orthogonal complement of $\mathbf{v}$ for any choice of vectors $\mathbf{x}$ and $\mathbf{v}$.

10. Let $\mathbf{x}$ be an eigenvector of the matrix A with eigenvalue 1, and let $\mathbf{y}$ be an eigenvector of A with eigenvalue 2. Moreover, assume that $\|\mathbf{x}\| = 1 = \|\mathbf{y}\|$ and that $\langle \mathbf{x}, \mathbf{y}\rangle = 0$.
    (a) Compute $\|\mathbf{v}\|$ for $\mathbf{v} = \mathbf{x} + 2\mathbf{y}$.
    (b) Compute $\|A\mathbf{v}\|$, with $\mathbf{v}$ as in (a).

11. Find an orthonormal basis of the orthogonal complement of the vector $[1,1,2]$ in $\mathbf{R}^3$.