

1. Compute the following double integrals:

(a)  $\int_1^2 \int_0^1 (1 + 4xy) dx dy$

(b) Integrate the function  $f(x, y) = 2 - x - 2y$  over the region  $D$  bounded by the line  $y = x - 1$  and by the parabola  $y^2 = 2x + 6$ . (hint: compute the intersections between the line and the parabola, and draw a picture of the region  $D$ ).

2. Compute the area of the following surfaces:

(a) The part of the plane  $2x + 5y + z = 10$  which lies in the first octant  $x \geq 0$ ,  $y \geq 0$  and  $z \geq 0$ .

(b) The part of the paraboloid  $z = 4 - x^2 - y^2$  that lies above the  $xy$ -plane.

(c) The part of the hyperbolic paraboloid  $z = y^2 - x^2$  that lies between the cylinders  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .

(d) The part of the sphere  $x^2 + y^2 + z^2 = 4$  which lies above the cone  $z^2 = x^2 + y^2$ .

3. Use the given transformation to evaluate the following integral:

(a)  $\iint_R \ln(x^2 + y^2) dA$ , where  $R$  is the region between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$  using polar coordinates  $x = \rho \cos \theta$  and  $y = \rho \sin \theta$ .

(b)  $\iint_R x dA$ , where  $R$  is the region bounded by the ellipse  $9x^2 + 4y^2 = 36$  with  $x = 2u$  and  $y = 3v$ .

Important formulas:

Surface area of surface given by parametrization  $\Phi(u, v)$  with  $(u, v)$  in a given domain  $D$ :

$$area = \int \int_D |\Phi_u \times \Phi_v| dudv.$$

Integral of a *function*  $f$  over a parametrized surface  $S$ :

$$\int \int_S f(x, y, z) dS = \int \int_D f(\Phi(u, v)) |\Phi_u \times \Phi_v| dudv.$$

Integral of a *vector field*  $\mathbf{F}$  over a parametrized surface  $S$ :

$$\int \int_S \mathbf{F}(x, y, z) d\mathbf{S} = \int \int_D \mathbf{F}(\Phi(u, v)) \cdot (\Phi_u \times \Phi_v) dudv.$$

Change of variable formula: See Section 15.6 (page 880) in the Shenk hand-out (downloadable as a pdf file) listed under 'Texts' on the course webpage, or look it up in Stewart, Section 15.9.