5.2.11 The Nullspace of $A - \lambda I = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$ is spanned by the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Thus, the eigenspace corresponding to λ is one dimensional.

$\dot{x} = Ax \Rightarrow \dot{y} = \lambda y$. So $y = C_2 e^{\lambda t}$ for some C_2.

$\dot{x} = \lambda x + by$ becomes $\dot{x} = \lambda x + bC_2 e^{\lambda t}$.

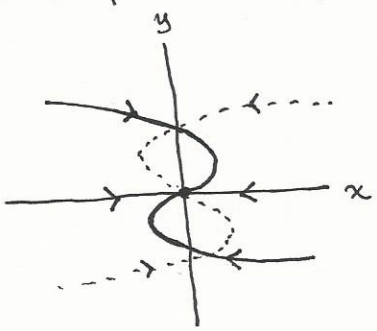
This has homogeneous solution $C_1 e^{\lambda t}$,

and particular solution $bC_2 e^{\lambda t} \cdot t$.

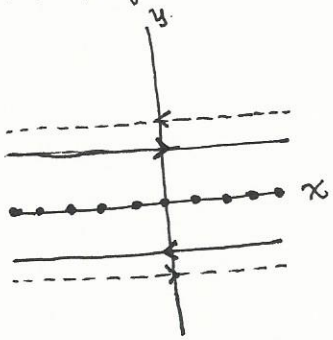
Hence, $\dot{x} = Ax$ has general solution

$$\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{\lambda t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + C_2 e^{\lambda t} \begin{bmatrix} bt \\ 1 \end{bmatrix}.$$

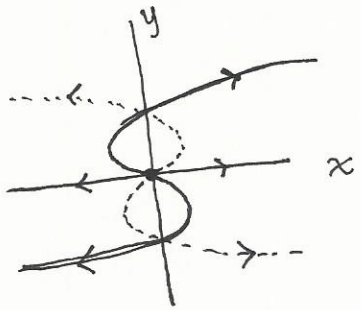
Phase portrait: [— indicates trajectories where $b>0$, ---- indicates trajectories where $b<0$]



$\lambda < 0$



$\lambda = 0$

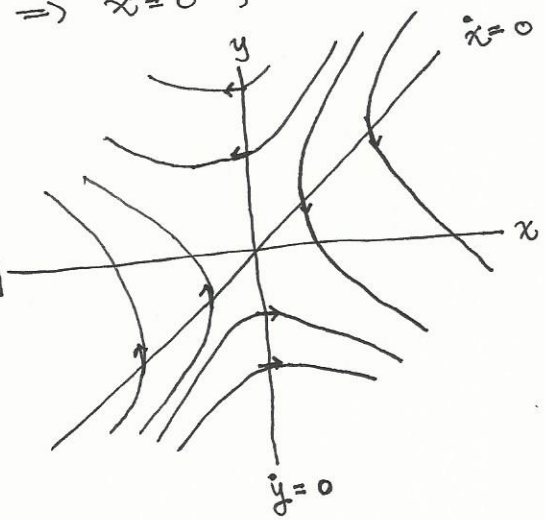


$\lambda > 0$

[+1 for dim. of eigenspace; +2 for general sol.; +2 for phase portraits]

6.1.1 $\dot{x} = x - y = 0 \Rightarrow x = y$ } fixed pt: $x = y = 0$

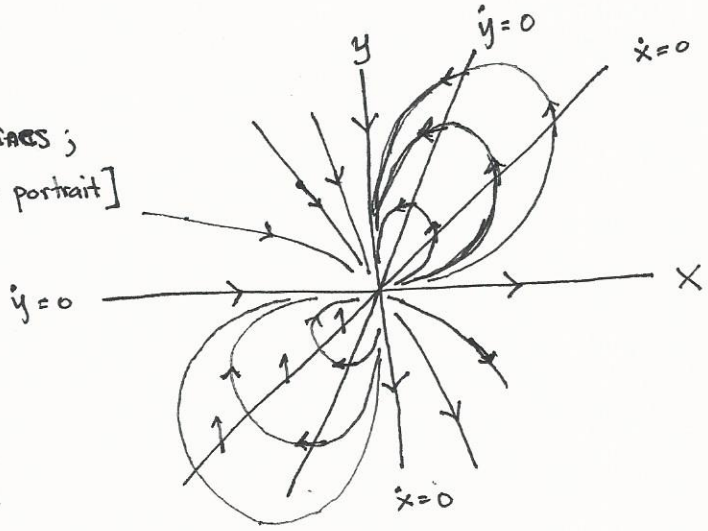
$\dot{y} = 1 - e^x = 0 \Rightarrow x = 0$ }



[+1 for fixed pt;
+2 for $\dot{x}=0$, $\dot{y}=0$ nullclines;
+2 for plausible phase portrait]

6.1.3 $\dot{x} = x(x-y) = 0 \Rightarrow x=0$ or $x=y$

$\dot{y} = y(2x-y) = 0 \Rightarrow y=0$ or $y=2x$

fixed pt: $x=y=0$

[+1 for fixed pt;
+2 for $\dot{x}=0$, $\dot{y}=0$ nullclines;
+2 for plausible phase portrait]