4.1.2 $\dot{\theta} = 1+2\cos\theta$. Fixed pts: $1+2\cos\theta=0 \iff \theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$

If $f(\theta) = 1+2\cos\theta$, then $f'(\theta) = -2\sin\theta$. $f'(\frac{2\pi}{3}) < 0 \Rightarrow \theta = \frac{2\pi}{3}$ is stable

$f'(\frac{4\pi}{3}) > 0 \Rightarrow \theta = \frac{4\pi}{3}$ is unstable

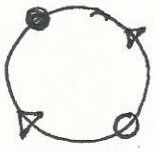
Phase Portrait:

4.1.5 $\dot{\theta} = \sin\theta + \cos\theta$. Fixed pts: $\sin\theta = -\cos\theta \iff \theta = \frac{3\pi}{4}$ or $\theta = \frac{7\pi}{4}$

If $f(\theta) = \sin\theta + \cos\theta$, then $f'(\theta) = \cos\theta - \sin\theta$. $f'(\frac{3\pi}{4}) < 0 \Rightarrow \theta = \frac{3\pi}{4}$ is stable.

$f'(\frac{7\pi}{4}) > 0 \Rightarrow \theta = \frac{7\pi}{4}$ is unstable.

Phase Portrait:



4.3.2 (a) $\theta = 2\tan^{-1}u$, $d\theta = \frac{2\,du}{1+u^2}$

(b)

$\sin\theta = \sin(2\cdot\frac{\theta}{2}) = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= \frac{2u}{1+u^2}$

(c) $\lim_{\theta\to\pm\pi} u(\theta) = \lim_{\theta\to\pm\pi} \tan\frac{\theta}{2} = \pm\infty$

(d) $T = \int_{-\infty}^{\infty} \frac{2\,du/(1+u^2)}{\omega - 2au/(1+u^2)} = 2\int_{-\infty}^{\infty} \frac{du}{\omega u^2 - 2au + \omega}$

(e) $\omega u^2 - 2au + \omega = \omega(u - a/\omega)^2 + \omega - a^2/\omega$.

Then $T = 2\omega^{-1}\int_{-\infty}^{\infty} \frac{du}{(u-a/\omega)^2 + 1 - (a/\omega)^2}$. If $x = u - a/\omega$ and $r = 1-(a/\omega)^2$,

then $T = 2\omega^{-1}\int_{-\infty}^{\infty} \frac{dx}{x^2+r}$. Now use 4.3.1.