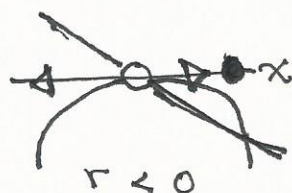
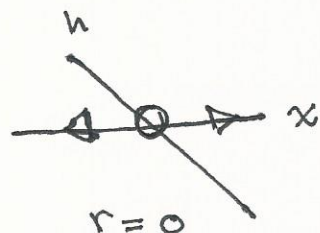
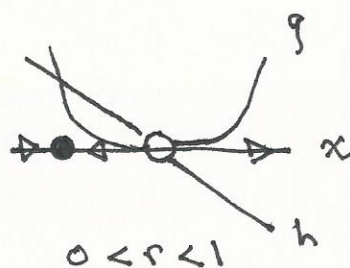
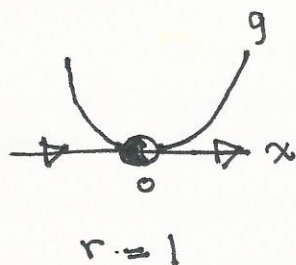
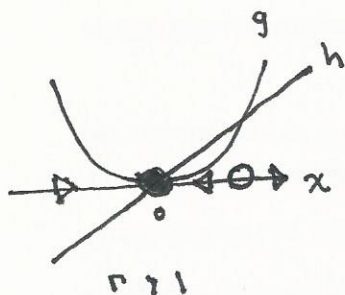


3.2.3

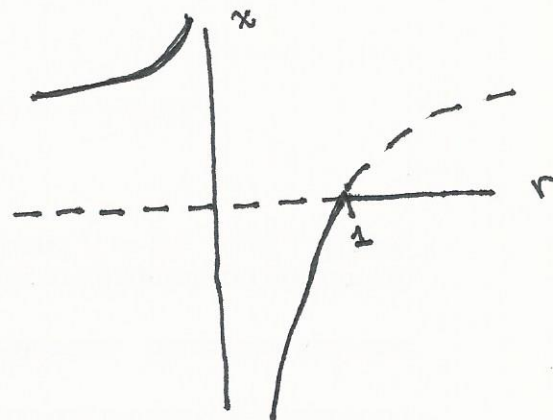
$$\dot{x} = x - rx(1-x) = \underbrace{rx^2}_g - \underbrace{(r-1)x}_h$$

→ Fixed points: $x=0$, $x=1-\frac{1}{r}$



Transcritical bifurcation at $r=1$.

Bifurcation Diagram:



• Vector Fields [2 pts]

→ 1 pt for 3/5 graphs

→ 2 pt for 5/5 graphs

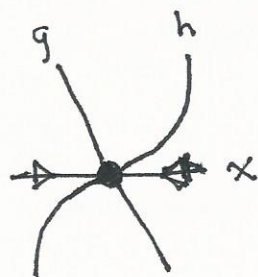
• Bifurcation at $r=1$ [1 pt]

• Bifurcation Diagram [2 pts]

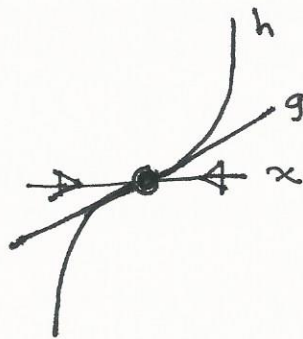
→ 1 pt for correct graph ($x=0$, $x=1-\frac{1}{r}$)

→ 1 pt for correct stability

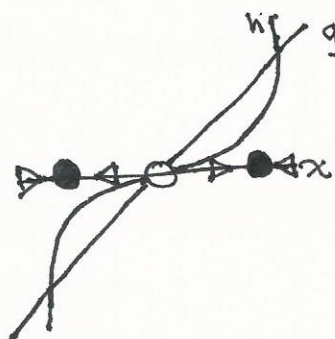
3.4.2 $\dot{x} = \underbrace{rx}_g - \underbrace{\sinh x}_h$



$r < 1$



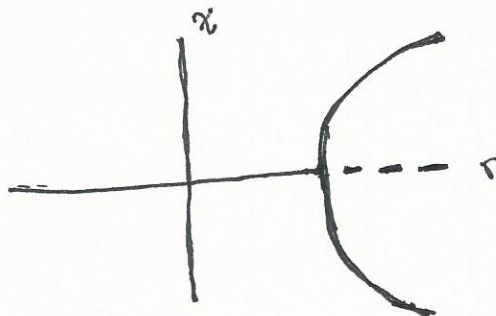
$r = 1$



$r > 1$

Bifurcation Diagram:

Supercritical Pitchfork
Bifurcation at $r = 1$



- Vector Fields [1 pt]
- Supercritical pitchfork [1 pt] at $r = 1$ [1 pt]
- Bifurcation Diagram [2 pts]
 - 1 pt for correct graph
 - 1 pt for correct stability.

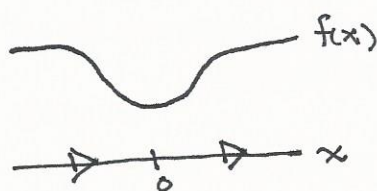
3.4.7

$$\dot{x} = 5 - r e^{x^2} = f(x)$$

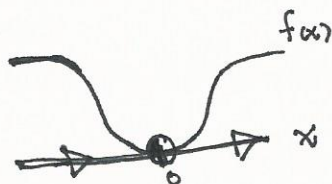
$$\rightarrow \text{Fixed pts: } x = \pm \left(\log\left(\frac{r}{5}\right) \right)^{1/2}$$

No fixed pt unless $\log\left(\frac{r}{5}\right) \geq 0$

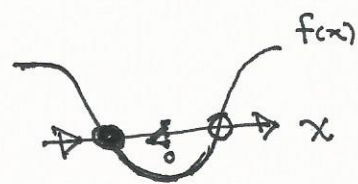
$$\Leftrightarrow r \geq 5$$



$$r < 5$$



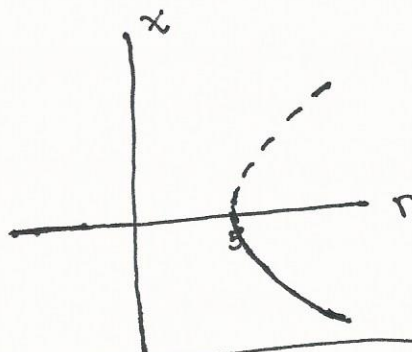
$$r = 5$$



$$r > 5$$

Bifurcation Diagram:

Saddle-Node bifurcation
at $r = 5$



- Saddle-Node bifurcation [2 pts] at $r = 5$ [1 pt]
- Bifurcation diagram [2 pts]
 - $\rightarrow$ 1 pt for correct graph
 - $\rightarrow$ 1 pt for correct stability