Part I

Short Design Course

Chapter 1

A method for solving system design problems

This chapter outlines our approach to solving design problems. Section 1.1 gives basic facts about rational functions, which are the single most important type of functions in practical system design. The basic system and functions considered in this book are introduced in Section 1.2 and Section 1.3. A control design problem is presented in 1.4. A description of the method used in this book to solve control problems is in Section 1.5.

1.1 Rational functions

A rational function can be thought of as a function on the imaginary axis $F(j\omega)$, or as a function on the complex plane, where the notation $F(s)$ is used. If $N(s)$ and $D(s)$ are polynomials, then the rational function $F(s) = \frac{N(s)}{D(s)}$ has *relative degree* given by

$$d(F(s)) := degree(D(s)) - degree(N(s))$$

The function F is called *proper* if $d(F) \geq 0$, and *strictly proper* if $d(F) > 0$.

A function F is said to be *bounded* (on the imaginary axis) if there exists a constant $M > 0$ such that

$$|F(j\omega)| \ \leq \ M, \ \ \forall \omega \in R$$

The smallest of such M is the number $||F||_{\infty} = \sup_{\omega \in R} |F(j\omega)|$, called the *supremum of* $|F(j\omega)|$ (See Fig. 1.1).

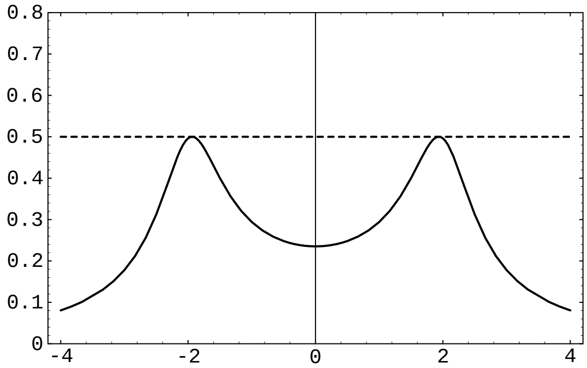

Figure 1.1: Plot of $|f(j\omega)|$ vs. frequency ω, where $f(s) = 1/((s-.5)^2+4)$. The dotted line marks $||f||_\infty$.

A rational function F is bounded if and only if it is proper and has no poles on the imaginary axis; the proof of this is left as an exercise. The value $||F||_\infty$ is not always attained, as the example below illustrates.

Example. The function $F(s) = s^2/(s^2-10.)$ is bounded, since $|F(j\omega)| \leq 1 \ \forall\omega$. However, for this example there is no single finite frequency ω_0 at which $|F(j\omega_0)| = 1$. In other words, the supremum is not attained.

Functions $F(s)$ that satisfy the equality

$$F(s) = \overline{F(\overline{s})}, \tag{1.1}$$

for all s, are sometimes referred to as *real* (on the real axis), and are very important in engineering. They arise as the Laplace transform of real valued functions of time. On the imaginary axis equality (1.1) becomes

$$F(j\omega) = \overline{F(-j\omega)} \tag{1.2}$$

All functions that appear in this book have this property. One can show that a rational function is real if and only if the coefficients are real numbers.

A rational function is *stable* if every pole of the function has negative real part, and the relative degree is not positive.

The usual convention is to denote the set of all rational functions that are stable and real by $\mathcal{RH}^\infty$. Note that functions in $\mathcal{RH}^\infty$ can be described as proper rational functions with real coefficients and with no poles in the closed Right-Hand-Plane RHP [1].

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[1] Some of the concepts introduced here apply to other (non rational) functions as

1.2 The closed loop system $\mathcal{S}$

The basic system we consider is a linear, time invariant, finite dimensional system in the frequency domain description depicted in Fig. 1.2. The functions $P(s)$ and $C(s)$ are proper, real rational, functions of s. They are called the plant and the compensator respectively. Fig. 1.2 is called the closed loop system and denoted by $\mathcal{S}$. *We assume that the plant in Fig. 1.2 is given and can not be modified, and that many choices of the compensator are possible.*

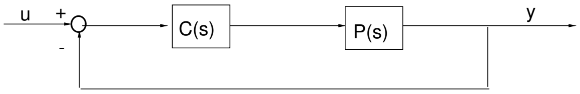

Figure 1.2: The closed loop system $\mathcal{S}$.

Besides $P(s)$ and $C(s)$, there are other functions commonly associated with the closed loop system $\mathcal{S}$, which we now derive. Begin with Fig. 1.2 to obtain the equations

$$\begin{array}{rcl} y(s) & = & P(s)C(s)e(s) \\ e(s) & = & u(s) - y(s) \end{array} \tag{1.3}$$

Combine both equations in (1.3) to obtain

$$y(s) = P(s)C(s)u(s) - P(s)C(s)y(s)$$

Solve for $y(s)$ and obtain

$$y(s) = (1 + P(s)C(s))^{-1} P(s)C(s)u(s) \tag{1.4}$$

Equation (1.4) has in it the closed loop transfer function of $\mathcal{S}$, defined as

$$T = \frac{PC}{1 + PC}. \tag{1.5}$$

Note that the variable s has been suppressed in (1.5) to simplify notation. Other key functions are:

- The sensitivity function $S := (1 + PC)^{-1}$.

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well. For example, a continuous function $F(j\omega)$ may be bounded or may satisfy (1.2). However, common mathematical techniques that work with rational functions do not work with more general types of functions (see [Fo]).

- The tracking error for input u, Su.
- The closed loop compensator $Q := CS$.
- The open loop transfer function $L := PC$.

1.3 Designable transfer function

Roughly speaking, to design a system one must choose all of the system's parts that have not been given in advance. This is done according to some criteria or requirements.

The most obvious way to select all the parts of system $\mathcal{S}$ is to specify C. Doing this determines the closed loop system $\mathcal{S}$ completely, in the sense that all its functions T, L, S, Q, etc. can be written in terms of the given part P and our selection for C.

In this book we will use another way to select a closed loop system $\mathcal{S}$. Think of T as a variable[2]. If T is specified, then one can calculate C by solving in (1.5):

$$C = \frac{T}{(1-T)P} \tag{1.6}$$

Now we see that it is possible to write C and every function in terms of T and the given part P. That is, to determine the closed loop system $\mathcal{S}$ one has only to pick a particular value for T. Under these circumstances we refer to T as a *designable transfer function.*

The designable transfer function cannot take an arbitrary form. There are many properties that the physics or mathematics of the system dictate. There is a set $\mathcal{I}$ of admissible functions to which the designable T must belong. For example, $\mathcal{I}$ contains functions that are continuous on the imaginary axis and uniformly bounded there. Clearly, the set $\mathcal{I}$ must be defined before the design process begins. In this book the set $\mathcal{I}$ is a subset of $\mathcal{RH}^\infty$.

1.4 A system design problem

We now formalize the generic problem we attempt to solve in this book.

The designer rejects or accepts a system by comparing the actual characteristics of the system to a set of requirements that describes what is

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[2] This has the advantage that design specifications are usually presented in terms of the closed loop transfer function T. It also has a mathematical advantage which is important to the optimization algorithms which underlie our method.

expected from the system. The two main types of requirements we consider are internal stability and performance.

The system $\mathcal{S}$ is internally stable if its closed loop transfer function T is stable and there is no cancellation of unstable poles and zeros in the product PC. Internal stability is treated in Chapter 2.

In the simplest possible terms, a performance requirement is an inequality in terms of the designable closed loop transfer function T. An example is

$$|T(j\omega)| < 0.707, \qquad for\ w > 1.0$$

Performance requirements are discussed in Chapter 3 and Chapter 6.

We state the design problem as follows:

Design	Given a plant P, a set of performance requirements $\mathcal{P}$ and a set of feasible functions $\mathcal{I} \subset \mathcal{RH}^{\infty}$, determine if there exists $T \in \mathcal{I}$ which makes the closed loop system $\mathcal{S}$ internally stable and satisfy all the performance requirements in $\mathcal{P}$. If the answer is yes, find one such T.

1.5 The method

A solution to *Design* is found by the following sequence of steps:

I. Obtain a mathematical description of the performance and internal stability requirements.

Some designable transfer functions T give rise to internally stable systems. These T can be described with a formula (see Chapter 2).

Each performance requirement is an inequality in terms of the designable transfer function T. In Chapter 3 we present a list of frequency domain performance requirements from which the designer can choose to formulate problems. This list is expanded in Chapters 4 and 5 by adding time-domain requirements.

II. Obtain a performance function and a performance index.

The mathematical expressions for the performance requirements are combined to form a *performance function* $\Gamma(\omega, T(j\omega))$, a positive valued function of T and ω. In many cases the performance function is defined in such a way that a designable T satisfies the requirements if and only if

$$\Gamma(\omega, T(j\omega)) \leq 1, \qquad (\forall \omega) \tag{1.7}$$

By considering the worst case performance over all frequencies, a *performance index* that depends on the closed loop transfer function T is obtained:

$$\gamma(T) = \sup_{\omega} \Gamma(\omega, T(j\omega)) \tag{1.8}$$

The performance index is a single number that gives a measure of goodness of a choice of T. In other words it is a cost function; the smaller $\gamma(T)$, the better the choice of T. Several ways to form Γ from the requirements are discussed in Chapters 4 and 5.

III. Minimize the performance index over all T that correspond to an internally stable system $\mathcal{S}$.

This is the step that requires a special purpose computer optimization program. Minimizing the performance index over all possible T produces two outputs:

- A yes or no answer to the question of whether there exist closed loop systems that meet all the requirements.
- An optimal designable transfer function T^*.

The function T^* is among the closed loop transfer functions that satisfy internal stability requirements the best, in terms of overall performance as measured by the index. Optimization (minimizing the performance index) is treated in Chapter 4.

If it is determined in step III that there are solutions to *Design*, the optimal designable transfer function T^* can be selected to produce simulations, physical realization, etc. However, in some cases it is more appropriate to restate the problem by selecting some of the requirements to make them more stringent (this allows one to put special emphasis on these requirements). This action results in a new Design problem, which is subsequently solved with steps I, II and III.

If the result of step III is that no solutions to *Design* exist, the engineer is confronted with two possible paths. One is to redefine the problem completely by changing the model. The other is to reexamine the requirements to determine which of them can be relaxed. In this case, a problem is formulated with the new set of requirements, and then solved with steps I, II and III.

Since in practice it is common to repeat steps I, II and III several times in the manner described above, we add another step to our list. With step IV below one can treat a sequence of *Design* problems.

IV. If many satisfactory closed loop systems exist or none exist, tighten or loosen the specifications accordingly and go to I. Stop if this process can not be carried further.

The modification of requirements is illustrated with practical examples in Chapter 5.

The mathematics used here produce T^* and C^* given by a set of values on the $j\omega$ axis. In most instances the designer will want to represent C^* as a rational function. In some cases, it is desirable that this rational function has low order. These two objectives can be accomplished with the techniques provided by the subjects of system identification and model reduction. While these subjects are not treated in this book[3], the software package OPTDesign has functions for doing (stable or otherwise) rational fits of data, in either the supremum norm or the ℓ^2 norm.

We finish this chapter with a caveat. It may be desirable when doing system design to select compensators that are stable. For example, this would be the case if the engineer wants to build and test the compensator as an independent unit. In its current state, the theory and computational methods of H^∞ optimization do not handle stability of compensators. Thus optimal compensators C^* produced by the methods used here are not necessarily stable (see [DFT], page 79).

1.6 Exercises

1. Exercises on rational functions.

a. Verify that the properties of being real, bounded, stable, and proper are preserved under the following operations: addition of two rational functions, multiplication of rational functions, multiplication of a rational function by a Real number.

b. Prove that a rational function is real if and only if the coefficients are real numbers.

c. Prove that a rational function F is bounded if and only if it is proper and has no poles on the imaginary axis.

2. Use formula (1.6 in verifying the following relations:

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[3] The reader is referred to [SZ] and [GKL] for theoretical aspects of system identification and model reduction. See also [Be] and [Th].

a. $\frac{dT}{dP} = S\frac{T}{P}$

b. $S + T = 1$

c. $L = \frac{T}{1-T}$

3. Write all functions L, C, S, Q in terms of P and T.

4. Consider a closed loop system $\mathcal{S}$ where T is stable and no RHP pole-zero cancelation occurs in PC. Prove that in this case,

a. if s_0 is a zero of P in the RHP, then $T(s_0) = 0$.

b. if s_1 is a pole of P in the RHP, then $T(s_1) = 1$.

5. Write as a frequency domain performance requirement the relation Rewrite the relation

$$|S(j\omega)| < 0.5 \qquad for\ 0 \leq \omega < 1$$

in terms of $T(j\omega)$.

1.7 References

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[Th] L. N. Trefethen. Matlab programs for CF approximation. Numerical Analysis Report 86-3. Dep. of Math. MIT. June 1986.

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Chapter 2

Internal Stability

This chapter is an introduction to internal stability of systems. The main concept is discussed in Section 2.1, where it is illustrated with examples. Internal stability is closely connected to a mathematical topic called interpolation, and that is the subject of Section (8). We conclude the chapter with a section on control systems for a given stable plant. The chapter gives the student quick overview of internal stability, as well as enough background to treat design problems of certain types. The discussion of internal stability in full generality is left to the more advanced student and is presented in Chapter 8.

2.1 Control and stability.

Consider the closed loop system $\mathcal{S}$ depicted below.

Figure 2.1: The closed loop system $\mathcal{S}$.

In Fig. 2.1, P and C are proper real rational functions. Given a plant P, by choosing an appropriate compensator C one can obtain a transfer function T for the closed loop system that is stable, i.e., that has no poles in the closed RHP. One way to achieve this is to choose C that cancels RHP poles and zeros of P. However, any RHP pole-zero cancellation in the product PC is highly undesirable, because small uncertainties in the plant P or in the compensator's construction lead to a radical change in the behavior of the closed loop system $\mathcal{S}$. We now illustrate this point with an example.

Example. For the given plant $P(s) = 1/(s-1)$, consider the compensators $C_1(s) = (s-1)/(s+1)$ and $C_2(s) = 2$. Observe that a RHP zero of the compensator C_1 cancels a RHP pole of the plant when the product $P(s)C_1(s)$ is formed:

$$P(s)C_1(s) = \frac{1}{s-1}\frac{s-1}{s+1} = \frac{1}{s+1}. \tag{2.1}$$

That the closed loop transfer function T_1 associated with C_1 is stable is clear from the following calculation:

$$T_1(s) = \frac{P(s)C_1(s)}{1+P(s)C_1(s)} = \frac{\frac{1}{s+1}}{1+\frac{1}{s+1}} = \frac{1}{s+2}$$

Now consider a small perturbation of the plant corresponding to a change in the location of the pole. Thus we now have the plant

$$P^\delta(s) = \frac{1}{s-1+\delta} \tag{2.2}$$

where δ is a real number with small absolute value. For this plant and the compensator C_1, the closed loop transfer function is

$$T_1^\delta(s) = \frac{P^\delta(s)C_1(s)}{1+P^\delta(s)C_1(s)} = \frac{\frac{s-1}{(s+1)(s-1+\delta)}}{1+\frac{s-1}{(s+1)(s-1+\delta)}} = \frac{s-1}{(s+1)(s-1+\delta)+(s-1)}$$

The function T_1^δ has an unstable pole at $s = s_\delta$ for all small values of δ, except for $\delta = 0$. Fig. 2.2 shows the plot of s_δ. Here

$$s_\delta = \frac{1}{2}\left(-(1+\delta)+\sqrt{(1+\delta)^2-4(\delta-2)}\right).$$

On the other hand, for the plant P^δ and compensator C_2 the closed loop transfer function is

$$T_2^\delta(s) = \frac{P^\delta(s)C_2(s)}{1+P^\delta(s)C_2(s)} = \frac{\frac{2}{s-1+\delta}}{1+\frac{2}{s-1+\delta}} = \frac{2}{s+1+\delta}.$$

Note that T_2^δ is stable for all small δ, while T_1^δ is not.

In this example we see that cancellation of a RHP pole of P with a zero of C produces an undesirable system, since the latter becomes unstable under

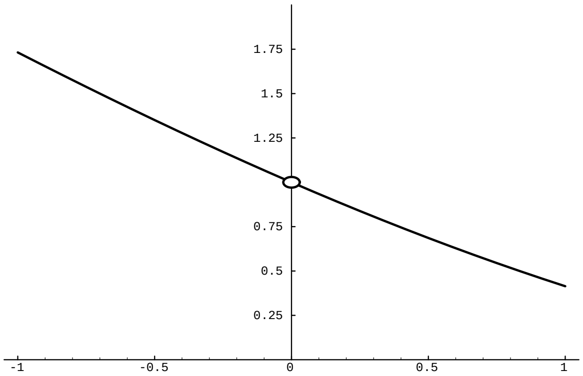

Figure 2.2: Plot of the unstable pole s_δ of P^δ as a function of δ. The parameter δ is on the horizontal axis. The point $(0, 1)$ is omited, since for $\delta = 0$ there is no unstable pole in P^δ.

small perturbations of the plant. A similar phenomenon occurs when a RHP zero of the plant is cancelled with a pole of the compensator (Exercise 1). Thus stability of T, sometimes referred to as *external stability* of the closed loop system, is not sufficient to guarantee a satisfactory closed loop system $\mathcal{S}$, although it is a necessary ingredient in the practical design of systems.

The closed loop transfer function describes external, or input-output, behavior of the system. A more satisfying concept of stability is that of *internal stability.*

Definition 1 *The system $\mathcal{S}$ is internally stable if*

i. *The closed loop transfer function T is in $\mathcal{RH}^\infty$.*

ii. *A RHP pole of the plant is not cancelled by a RHP zero of the compensator.*

iii. *A RHP zero of the plant is not cancelled by a RHP pole of the compensator.*

Here $\mathcal{RH}^\infty$ is the linear space of all proper rational functions that are stable (poles off the closed *RHP*) and real (real coefficients).

The fundamental problem we treat in this chapter is, given a plant P in a certain class, find a description of *all* the internally stable systems $\mathcal{S}$ with P as plant. The same problem is discussed in Chapter 8 for any plant P.

There are many ways in which one can present an answer to this problem, all of which are mathematically equivalent. In this book we choose one that consists in writing *a formula* that is (and must be) satisfied by all designable closed loop transfer functions T of these systems. The reasons for choosing this particular approach are technical rather than physical: we have available a procedure that uses this formula to give an answer to system design problems.

To derive formulas for internally stable systems we need first the concept of interpolation condition which is presented in the next section.

2.2 Interpolation.

A basic result from elementary analysis states that given a point s_0 in the complex plane and a polynomial f, then

$$f(s_0) = 0 \tag{2.3}$$

if and only if one can write

$$f(s) = (s - s_0)h(s) \tag{2.4}$$

where h is some polynomial.

The equality (2.3) is called an interpolation condition. One can prove that (2.4) gives a formula or parameterization of all functions f satisfying (2.3). In this chapter the functions f are rational functions, rather than just polynomials.

To illustrate the role of interpolation in internal stability, we consider the case of a system $\mathcal{S}$ with a plant P that has a simple RHP pole at $s = s_0$. In the discussion below the following interpolation condition will be needed:

$$T(s_0) = 1 \tag{2.5}$$

Claim: If s_0 is a simple RHP pole of P and T satisfies (2.5), then there is no pole zero cancellation in the product $P(s)C(s)$ at $s = s_0$.
Proof: If there is cancellation, then $P(s_0)C(s_0)$ is finite, and this implies that

$$T(s_0) = \frac{P(s_0)C(s_0)}{1 + P(s_0)C(s_0)} \neq 1.$$

This proves the claim.

Claim: If s_0 is a simple RHP pole of P and if there is no RHP pole-zero cancellation in PC at $s = s_0$, then T satisfies (2.5).

Proof: If there is no cancellation in the product $P(s)C(s)$ at $s = s_0$, then $P(s)C(s)$ has an order $n_0 \geq 0$ pole at $s = s_0$. Let

$$\ell := \lim_{s \to s_0} (s - s_0)^{n_0} P(s)C(s)$$

Then ℓ is a number, and

$$\begin{aligned} T(s_0) &= \lim_{s \to s_0} \frac{P(s)C(s)}{1+P(s)C(s)} \\ &= \lim_{s \to s_0} \frac{(s-s_0)^{n_0} P(s)C(s)}{(s-s_0)^{n_0}+(s-s_0)^{n_0}P(s)C(s)} = \frac{\ell}{0+\ell} = 1. \end{aligned}$$

More is true:

Proposition 1 *Let $\mathcal{S}$ be a system with plant P and closed loop transfer function T.*

- *If s_0 is a simple RHP pole of P, then pole-zero cancellation in the product PC does not occur at $s = s_0$ if and only if $T(s_0) = 1$.*
- *If s_0 is a simple RHP zero of P, then pole-zero cancellation in the product PC does not occur at $s = s_0$ if and only if $T(s_0) = 0$.*

One consequence of Proposition 1 is that for certain systems, internal stability is described in terms of a set $\mathcal{J}$ of interpolation conditions such as $T(s_0) = 0$ and $T(s_0) = 1$.

The good thing about interpolation conditions is that one can always write down a formula for all functions T in $\mathcal{RH}^\infty$ that satisfy given interpolation conditions. Such a formula looks like:

$$T(s) = A(s) + B(s)\, T_1(s) \tag{2.6}$$

where A and B are certain functions in RH^∞ that can be determined from $\mathcal{J}$, and T_1 could be any function in RH^∞. Thus as T_1 sweeps $\mathcal{RH}^\infty$, we have T sweeping all functions in $\mathcal{RH}^\infty$ meeting $\mathcal{J}$.

One can show that a system is internally stable if T satisfies (2.6) for the interpolation conditions $\mathcal{J}$ corresponding to poles and zeros of the plant P located in the RHP. A complete discussion of this can be found in Chapter (8), but for now we restrict our study to the case where the plant is stable.

2.3 Systems with a stable plant

Systems $\mathcal{S}$ with a RHP-stable plant are a common type of system occurring in practical control systems. These are also simple enough to provide a good introduction to a study of internal stability.

We now try to answer the question, given a stable plant P, what systems $\mathcal{S}$ have P as plant and are internally stable?

To give an answer we will find a formula for the closed loop transfer functions T that come from such systems. Here is a restatement of the question above: Given a stable plant P, describe all possible closed loop transfer functions T that come from an internally stable system $\mathcal{S}$ with plant P. The answer we seek is in the following theorem.

Theorem 1 *Let P be a strictly proper, RHP-stable plant for the system $\mathcal{S}$. Then the system $\mathcal{S}$ is internally stable if and only if there exists $T_1 \in RH^\infty$ such that $T = PT_1$*

Proof: If $\mathcal{S}$ is internally stable, then PC has the same zeros as P, including multiplicity. Thus the zeros of $T = PC/(1+PC)$ contain the zeros of P, and this implies that the function $T_1 := T/P$ has no RHP poles. The function T_1 is proper since the system $\mathcal{S}$ has proper C and $T_1 = C/(1+PC)$. Thus we have that T_1 is an element of RH^∞. This proves the only if side of the Theorem. Conversely, suppose that $T = PT_1$ for some $T_1 \in RH^\infty$. Note that the compensator is given by

$$C = \frac{T}{P(1-T)} = \frac{T_1}{1-PT_1}$$

It is clear from this formula and from P being strictly proper that C is proper. For C to have a pole at the same location $s = s_0$ as a RHP zero of P one must have $1 - P(s_0)T_1(s_0) = 0$, which is impossible since T_1 has no pole at s_0 and P is zero there. ■

Example. If the plant of an internally stable system is $P(s) = \frac{1}{(s+3.4)^2}$, then any stable closed loop transfer function T has the form

$$T(s) = \frac{1}{(s+3.4)^2} * T_1(s) \qquad (2.7)$$

where T_1 is some (or any) element of RH^∞. We note that T_1 may not be strictly proper. In fact, $d(T_1) \neq 0$ forces $d(C) = 0$ upon the system.

Note that we used the actual plant P in the formula (2.7). While this may be convenient, many other formulas for T are possible. For example, we can correctly write

$$T(s) = \frac{1}{(s+1)^2}\, T_1(s) \qquad \forall T_1 \in \mathcal{RH}^\infty \qquad (2.8)$$

to describe all T from internally stable $\mathcal{S}$.

Example. If now $P(s) = (s^2 - 4)/(s^4 + 2s^2 + 1)$ then P is stable and strictly proper, so for the system to be internally stable the closed loop transfer function must have the form

$$T(s) = \frac{s^2 - 4}{s^4 + 2s^2 + 1} T_1(s)$$

for some $T_1 \in RH^\infty$.

2.4 Exercises

1. Consider the family of plants given by

$$P^\delta(s) = (s - 1 + \delta)/(s + 1)$$

   where δ is a real constant. Show that

   a. for small $|\delta|$, the compensator $C_1(s) = 1/(s-1)$ and plant P^δ yields a closed loop transfer function T^δ that is stable if and only if $\delta = 0$.

   b. for all small $|\delta|$, the compensator $C_2(s) = (s+1)/(s+3)$ and plant P^δ yields a stable closed loop transfer function T^δ.

Chapter 3

Frequency domain performance requirements

This chapter treats frequency domain performance requirements. These have a convenient graphical interpretation in terms of disks. Section 3.1 reviews basic concepts and introduces disk inequalities. The most common performance requiurements for control systems are given in Section 3.2. Section 3.3 discusses disk inequalities arising from performance requirements. The first time reader may stop here; the topics covered at this point are enough to give basic tools to solve simple design problems, when used in conjunction with material from Chapter 2 and Chapter 4. The more interested reader may continue with Sections 3.4 and 3.5, which explain additional measures of performance and the corresponding disk inequalities.

3.1 Introduction

3.1.1 The closed loop system $\mathcal{S}$

Consider the closed loop system depicted in Fig. 3.1. In this system, $P(s)$ and $C(s)$ are proper real rational functions of s. We take the plant P to be a given rational function, and use the closed loop transfer function T to parameterize the closed loop systems $\mathcal{S}$ obtained with different compensators. Hence many systems $\mathcal{S}$ with given plant P are possible by letting the designable transfer function T take different values. Key functions are