/home/m262f99/A=B/Maple/sum2int1.mws

Math 262a, Fall 1999, Glenn Tesler
Creative Telescoping for multisums/integrals
10/27/99

```
> read DOUBLE_SUM_SINGLE_INTEGRAL:
  P:=1:
  F:=rf(al+1,n)*rf(be+1,n)/n!/rf(al+be+1,n)/t^(n+1)*
  (1+t)^(-al-be-1)*rf(al+be+1,2*(m+n1))/n1!/m!/rf(al+1,n1)
  /rf(be+1,m)*
  (t/(4*(1+t)^2))^(m+n1)*
  (1+x)^m*(1+y)^m*(1-x)^n1*(1-y)^n1;
  ORDER:=2:
  resh:=[al,be,x,y,n]:
  S1:=1/(t+1):
  S2:=(n1+al)*n1:
  S3:=m*(m+be):
  disorcon:=continuous:
  cer:=findope(P,F,x,t,n1,m,ORDER,resh,D_x,S1,S2,S3,disorc
  on);
```

*THIS IS VERSION 1.1 OF THE MAPLE PACKAGE multi1c2d FOR INTEGRAND/SUMMANDS DEPENDING OF ONE CONTINUOUS(INTEGRATIO\
N) VARIABLE*

AND TWO DISCRETE(SUMMATION) VARIABLES

THE PREOGRAM multi1c2d BASED ON THE PAPER BY:

*WILF AND ZEILBERGER:"AN ALGORITHMIC PROOF THEORY FOR (ORDINA\
RY AND 'q')*

*MULTISUMS/INTEGRAL HYPERGEOMETRIC IDENTITIES(Invent. Math. 108(19\
92)*

pp.575-633).

The primilary version of the program was written by Doron Zeilberger

Please report all bugs and comments to: zeilberg@math.temple.edu

or akalu@math.temple.edu

For a list of procedures, type:
?muti1c2d or help(muti1c2d)

For help with a specific procedure, type:
?procedure name or help(procedure name)

$$F := \Gamma(al+1+n)\,\Gamma(be+1+n)\,(1+t)^{(-al-be-1)}\,\Gamma(al+be+1+2\,m+2\,n1)\left(\frac{1}{4}\frac{t}{(1+t)^2}\right)^{(m+n1)}(1+x)^m\,(1+y)^m\,(1-x)^{n1}\,(1-y)^{n1}\Big/\left(n!\,\Gamma(al+be+1+n)\,t^{(n+1)}\,n1!\,m!\,\Gamma(al+1+n1)\,\Gamma(be+1+m)\right)$$

$$cer := n\,(al+be+1+n)+(-x\,al-x\,be-2\,x-al+be)\,D\_x-(x-1)\,(1+x)\,D\_x^2,\left[-\frac{t\,(al+be+1+n+t\,n-t\,n1-t\,m+m+n1)}{1+t},\,2\,\frac{(n1+al)\,n1}{x-1},\,-2\,\frac{m\,(m+be)}{1+x}\right]$$

```
> ope := cer[1]:
  R1 := cer[2][1]: R2 := cer[2][2]: R3 := cer[2][3]:
  writepap(P,F,x,y1,k1,k2,D_x,ope,R1,R2,R3,disorcon);
```

Theorem:

Let G(y1 , k1 , k2 , x) be

$$\Gamma(al+1+n)\,\Gamma(be+1+n)\,(1+t)^{(-al-be-1)}\,\Gamma(al+be+1+2\,m+2\,n1)\left(\frac{1}{4}\frac{t}{(1+t)^2}\right)^{(m+n1)}(1+x)^m\,(1+y)^m\,(1-x)^{n1}\,(1-y)^{n1}\Big/\left(n!\,\Gamma(al+be+1+n)\,t^{(n+1)}\,n1!\,m!\,\Gamma(al+1+n1)\,\Gamma(be+1+m)\right),$$

and a(x) be its integral w.r.t to y1 ,sum w.r.t. k2 k1 .

Let D_x be differentiation w.r.t. x .

The function a(x) satisfies the recurrence

$$(n\,(al+be+1+n)+(-x\,al-x\,be-2\,x-al+be)\,D\_x-(x-1)\,(1+x)\,D\_x^2)\,\mathrm{a}(x)=0.$$

Proof: It is routinely verifiable that

$$(n\,(al+be+1+n)+(-x\,al-x\,be-2\,x-al+be)\,D\_x-(x-1)\,(1+x)\,D\_x^2)$$

$$G(y1, k1, k2, x)$$

= D_ y1 (

$$-\frac{t\,(al + be + 1 + n + t\,n - t\,n1 - t\,m + m + n1)\,G(y1, k1, k2, x)}{1 + t}$$

)

+(E_ k1 -I)(

$$2\,\frac{(n1 + al)\,n1\,G(y1, k1, k2, x)}{x - 1}$$

+(E_ k2 -I)(

$$-2\,\frac{m\,(m + be)\,G(y1, k1, k2, x)}{1 + x}$$

),

and the result follows by integrating w.r.t
y1 , and summing w.r.t. k1 k2 .

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