Math 262a, Fall 1999, Glenn Tesler
Koepf's "Extended Gosper" algorithm
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```
> read `hsum.mpl`;
```

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```
> gosper(k*k!,k);
```

$$k!$$

```
> a1 := (k/2)*(k/2)!;
  gosper(a1,k);
```

$$a1 := \frac{1}{2}\,k\left(\frac{1}{2}\,k\right)!$$

```
Error, (in gosper) algorithm not applicable
```

```
> s1 := extended_gosper(a1,k);
```

$$s1 := \left(\frac{1}{2}\,k\right)! + \left(\frac{1}{2}\,k + \frac{1}{2}\right)!$$

Note: the above is not a hypergeometric term, but it is an antidifference of (k/2)*(k/2)! with a simple, closed formula. Let's check it:

```
> simplify((subs(k=k+1,s1)-s1) - a1);
```

$$0$$

So we conclude that

```
> Sum(a1,k=a..(b-1)) = subs(k=b,s1)-subs(k=a,s1);
```

$$\sum_{k=a}^{b-1}\left(\frac{1}{2}\,k\left(\frac{1}{2}\,k\right)!\right) = \left(\frac{1}{2}\,b\right)! + \left(\frac{1}{2}\,b + \frac{1}{2}\right)! - \left(\frac{1}{2}\,a\right)! - \left(\frac{1}{2}\,a + \frac{1}{2}\right)!$$

A variation: instead of computing s(k) s.t. s(k+1)-s(k)=a(k), we supply an integer j and compute s(k) s.t. s(k+j)-s(k)=a(k). This is called a j-fold antidifference.
   extended_gosper(f(k),k,j);

```
> extended_gosper((k/2)*(k/2)!,k,2);
```

$$\left(\frac{1}{2}\,k\right)!$$

```
>
>
```