Math 262a — Topics in Combinatorics — Fall 1999 — Glenn Tesler
Homework 6 — November 12, 1999

The homepage for the class is still

`http://math.ucsd.edu/~gptesler/math262`

but all the files have been moved out of my account to a special class account. To examine the worksheets from your euclid account, type

```
cd ~m262f99/KOEPF/worksheetsV.4
xmaple &
```

and then open a worksheet from "Open" in the "File" menu. The examples within the text are in worksheets `chap1.mws` – `chap13.mws`, while the exercises are in `exer2.mws` – `exer13.mws`. These are from Koepf's web site. The problem numbers are generally not marked within the worksheet. I also put in the maple worksheets for my answer keys (but not the TEX'd portions of the answer keys) as `hw1.mws`, etc.

If you want to make changes to the worksheets or do your own from scratch, copy `hsum.mpl` and `qsum.mpl` and the relevant worksheets to your own directory.

1. **Computer problems.** Koepf # 9.1, 9.3(a), 9.12, 9.13, 9.16
2. **Traditional proof: computational complexity of Hyper.** Koepf # 9.11
3. **Reduction of order for a recurrence equation.**
   (a) Consider the recurrence equation

   $$\sum_{i=0}^{d} p_i(n) f(n+i) = 0 \qquad \text{where } p_d(n) \text{ isn't identically } 0.$$

   Let $f(n) = u(n)$ be one nontrivial solution found by any method. To find more solutions, substitute $f(n) = u(n)v(n)$. Prove that this recurrence equation becomes an order $d-1$ recursion equation for $w(n) = \Delta v(n)$. Solving for $w(n)$ leads to another solution for $f(n)$:

   $$v(n) = v(a) + \sum_{k=a}^{n-1} w(k) \qquad f(n) = u(n) \sum_{k=a}^{n-1} w(k)$$

   for any suitable constant a.
   (There's also an inhomogeneous version of this.)

   (b) A=B, p. 165 # 4.

Many of the methods you learned for solving differential equations have counterparts for recurrence equations. These include series solutions, variation of parameters, and reduction of order. The most complete references I found for these and other methods are

[1] L. M. Milne-Thomson, *The Calculus of Finite Differences*, Macmillan and Co., Ltd., London, 1933.

[2] C. M. Bender and S. A. Orszag, *Advanced mathematical methods for scientists and engineers*, International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York, 1978.