Math 262a — Topics in Combinatorics — Fall 1999 — Glenn Tesler

Homework 5 — November 5, 1999

1. Koepf # 6.7(e). Prove it by the WZ method and also derive the companion identity, determining proper bounds for the parameters.
2. Koepf # 8.5(a,b), 8.7 [identity p. 128 (5.21)]; 11.7.
3. Use Sister Celine's algorithm as shown in class to compute $g(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$:
   (a) Let $F(x,k) = \frac{x^{2k+1}}{(2k+1)!}$. Find a mixed recurrence/differential equation of the form
   $$\sum_{(i,j)\in S} a_{ij}(x) {D_x}^i {E_k}^j F(x,k) = 0$$
   where $D_x = \frac{\partial}{\partial x}$, E_k is the shift operator in k, $a_{ij}(x)$ are functions of x, and S is a suitable finite set.

   (b) Sum the equation found in part (a) over k to get a differential equation for $g(x)$.

   (c) Solve the differential equation, making use of suitable initial conditions.
4. Koepf # 12.1.