Math 262a — Topics in Combinatorics — Fall 1999 — Glenn Tesler
Homework 4 — October 29, 1999

1. (From A=B p. 104) Consider

$$f(n) = \sum_{k=0}^{\lfloor n/3 \rfloor} 2^k \cdot \frac{n}{n-k} \binom{n-k}{2k}$$

   (a) Verify that

$$({E_n}^2+1)(E_n-2)F(n,k) = G(n,k+1) - G(n,k) \qquad \text{where } G(n,k) = -\frac{2^k \cdot n}{n-3k+3}\binom{n-k}{2k-2}$$

   (b) Use this to evaluate $f(n)$. Note that the k sum is not $-\infty \le k \le \infty$, so extra steps are required.

2. In Koepf's software we have commands
   - For creative telescoping:
     `sumrecursion(binomial(n,k),k,s(n));` or `zeilberger(binomial(n,k),k,s(n));`

$$-s(n+1) + 2s(n) = 0$$

     `zeilberger` either gives a first order recurrence or fails, while `sumrecursion` tries orders 1,2,3, ... ,`MAXORDER` (a global variable, default 5).
   - For Sister Celine's algorithm:
     `fasenmyer(binomial(n,k),k,s(n),J);` (where you must set J to some specific number 0, 1, 2, etc.)
     `fasenmyer(binomial(n,k),k,s(n),0);` fails.
     `fasenmyer(binomial(n,k),k,s(n),1);` gives the same recurrence as above.
   - For doing creative telescoping and solving the resulting recurrence, provided it's first order:
     `closedform(binomial(n,k),k,n);`

$$2^n$$

     `Closedform(binomial(n,k),k,n);`

$$\text{Hyperterm}([1], [], 2, n)$$

   Use Sister Celine's algorithm and the Creative Telescoping algorithm to find recurrences for $s(n) = \sum_k \binom{n}{k}^3$. The answers are very different. Explain how to reconcile them.

3. Koepf # 7.4, 7.7, 7.11 (Krawtchouk), 7.15, 7.19(d,e), 7.21, 7.24(b).