/home/m262f99/KOEPF/worksheetsV.4/hw1ansm.mws

Math 262a, Fall 1999, Glenn Tesler
Homework 1

```
> restart;
> read ‘hsum.mpl‘;
```


To get help on the package, type

```
> ?hsum
>
```

Problem 1. Koepf 2.11(b)

Term ratio:

```
> ratio((-1)^(k+1)/k*x^k, k);
```

$$-\frac{x\,k}{k+1}$$

whole sum:

```
> Sumtohyper((-1)^(k+1)/k*x^k,k);
```

$$x\,\mathrm{Hypergeom}([1, 1], [2], -x)$$

maple's built-in sum command

```
> sum((-1)^(k+1)/k*x^k,k); # indefinite sum
```

$$\sum_k \frac{(-1)^{(k+1)}\,x^k}{k}$$

```
> sum((-1)^(k+1)/k*x^k,k=1..infinity); # definite sum
```

$$\ln(1+x)$$

```
> sum((-1)^(k+1)/k*x^k,k=5..infinity);
```

$$\frac{1}{5}x^5\left(\frac{5}{4}\frac{1}{x}-\frac{5}{3}\frac{1}{x^2}+\frac{5}{2}\frac{1}{x^3}-\frac{5}{x^4}+5\,\frac{\ln(1+x)}{x^5}\right)$$

```
> simplify(");
```

$$\frac{1}{4}x^4-\frac{1}{3}x^3+\frac{1}{2}x^2-x+\ln(1+x)$$

```
>
>
>
```

Koepf 2.11(f)

```
> atanterm := (-1)^k/(2*k+1) * x^(2*k+1);
  subs(k=0,atanterm); # initial term
  ratio(atanterm,k);
```

```
Sumtohyper(atanterm,k);
```

$$atanterm := \frac{(-1)^k x^{(2k+1)}}{2k+1}$$

$$x$$

$$-\frac{x^2 (2k+1)}{2k+3}$$

$$x \text{ Hypergeom}\left(\left[\frac{1}{2}, 1\right], \left[\frac{3}{2}\right], -x^2\right)$$

maple's built-in sum

```
> sum(atanterm,k); # indefinite sum
  sum(atanterm,k=0..infinity); # definite sum
```

$$\sum_k \frac{(-1)^k x^{(2k+1)}}{2k+1}$$

$$\frac{1}{2} \frac{x \ln\left(\frac{1+\sqrt{-x^2}}{1-\sqrt{-x^2}}\right)}{\sqrt{-x^2}}$$

```
>
>
```

Koepf 2.9(d)

```
> dterm := binomial(n,k)*binomial(2*k,n);
  ratio(dterm,k);
  Sumtohyper(dterm,k);
```

$$dterm := \text{binomial}(n, k) \text{ binomial}(2k, n)$$

$$-2 \frac{(-n+k)(2k+1)}{(2k-n+1)(2k+2-n)}$$

$$\text{binomial}(0, n) \text{ Hypergeom}\left(\left[\frac{1}{2}, -n, 1\right], \left[1-\frac{1}{2}n, -\frac{1}{2}n+\frac{1}{2}\right], -1\right)$$

Note: the above answer assumes n is not a nonnegative integer. See the answer key for a complete answer.

```
> Sumtohyper(subs(n=5,dterm),k);
```

$$60 \text{ Hypergeom}\left(\left[-2, \frac{7}{2}\right], \left[\frac{3}{2}\right], -1\right)$$

```
>
> sum(dterm,n=0..infinity);
```

$$\text{binomial}(0, k)\ \text{hypergeom}([1, -2k], [-k+1], -1)$$

Here's Method II.

```
> dterm2 := subs(k=n-k,dterm);
  ratio(dterm2,k);
  Sumtohyper(dterm2,k);
```

$$dterm2 := \text{binomial}(n, n-k)\ \text{binomial}(2n-2k, n)$$

$$-\frac{1}{2}\frac{(2k-n)(2k+1-n)}{(-2n+2k+1)(k+1)}$$

$$\text{binomial}(2n, n)\ \text{Hypergeom}\left(\left[\frac{1}{2}-\frac{1}{2}n, -\frac{1}{2}n\right], \left[-n+\frac{1}{2}\right], \text{-}1\right)$$

```
>
```

Problems 2-3 aren't here.

Problem 4, Koepf 2.21

```
> read ‘qsum.mpl‘;
```

Copyright 1998, Harald Boeing & Wolfram Koepf

Konrad-Zuse-Zentrum Berlin

2.21(a)

```
> qsimplify(qpochhammer(a,q,infinity)/qpochhammer(a*q^n,q,
  infinity));
```

$$\text{qpochhammer}(a, q, n)$$

2.21(b) the symbol b stands for sqrt(a)

```
> qsimplify(qpochhammer(q*b,q,n)*qpochhammer(-q*b,q,n) /
  (qpochhammer(b,q,n)*qpochhammer(-b,q,n)));
```

$$\frac{(-1+b\,q^n)(1+b\,q^n)}{(-1+b)(1+b)}$$

```
> expand(numer("))/expand(denom("));
```

$$\frac{-1+b^2\,(q^n)^2}{-1+b^2}$$

2.21(c)

```
> qsimplify(qpochhammer(a,q,n)*qpochhammer(-a,q,n));
```

$$\text{qpochhammer}(a^2, q^2, n)$$

2.21(d)

```
> form1 := qpochhammer(a,q,n);
  form2 := qpochhammer(q^(1-n)/a,q,n);
  form2b := qsimplify(form2);
```

$$form1 := \text{qpochhammer}(a, q, n)$$

$$form2 := \mathrm{qpochhammer}\left(\frac{q^{(1-n)}}{a}, q, n\right)$$

$$form2b := \frac{q^{(1/2\,n)}\,(-1)^n\,\mathrm{qpochhammer}(a, q, n)}{q^{(1/2\,n^2)}\,a^n}$$

```
> solve(form2=form2b, {form1});
```

$$\left\{\mathrm{qpochhammer}(a, q, n) = \frac{q^{(1/2\,n^2)}\,a^n\,\mathrm{qpochhammer}\left(\frac{q^{(1-n)}}{a}, q, n\right)}{q^{(1/2\,n)}\,(-1)^n}\right\}$$

built-in maple command that doesn't know about q functions/simplifications, so it won't touch the qpochhammer's but will simplify the rest

```
> simplify(");
```

$$\{\mathrm{qpochhammer}(a, q, n) = (-1)^{(-n)}\,q^{(1/2\,n^2 - 1/2\,n)}\,a^n\,\mathrm{qpochhammer}\left(\frac{q^{(1-n)}}{a}, q, n\right)\}$$

```
>
```

Koepf 3.15(b)

```
> sum2qhyper(qbinomial(n,k,q)^2 * x^k, q, k);
```

$$\phi([q^{(-n)}, q^{(-n)}, 0, 0], [q], q, q^{(2\,n)}\,x)$$

```
> qratio(qbinomial(n,k,q)^2 * x^k, k);
```

$$\frac{(-q^k + q^n)^2\,x}{(-1 + q\,q^k)^2\,(q^k)^2}$$

```
>
```