
Instructions:

- No calculators, tablets, phones, or other electronic devices are allowed during this exam.
 - You may use one handwritten sheet of notes (8.5"×11" double-sided), but no books or other aids.
 - Fill in your Name and PID on this page.
 - Read each question carefully and answer each question completely.
 - Write your solutions clearly and legibly.
 - Show all of your work; no credit will be given for unsupported answers.
 - Circle your final answers.
 - Exams must be turned in promptly when time is called.
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0. (1 point) Carefully read and complete the instructions on this exam and any additional instructions announced during the exam.

1. Evaluate the iterated integral $\int_0^4 \int_{\sqrt{x}}^2 6 \cos(y^3) dy dx$.

Hint: Write it another way.

2. Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be nonzero vectors in $\mathbb{R}^3$. Set

$$\vec{v} = \vec{a} \times (\vec{b} \times \vec{c}) \quad \text{and} \quad \vec{w} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}.$$

For each of the following, provide a *brief* explanation of no more than two or three sentences.

- (a) Explain why $\vec{v}$ is orthogonal to $\vec{a}$.
- (b) Explain why $\vec{v}$ lies in the plane spanned by $\vec{b}$ and $\vec{c}$. That is, explain why $\vec{v}$ is orthogonal to the normal vector of the plane parallel to both $\vec{b}$ and $\vec{c}$.
- (c) Explain why $\vec{w}$ is also orthogonal to $\vec{a}$.
- (d) Explain why $\vec{w}$ also lies in the plane spanned by $\vec{b}$ and $\vec{c}$.
- (e) What is the relationship between $\vec{v}$ and $\vec{w}$, based on parts (a)–(d) above?

3. Let $D = \{ (x, y) : x^2 + y^2 \leq 4 \}$. Briefly explain why $\iint_D \frac{\sin^2(x)}{x^2 + y^2 + 4} dA \leq \pi$.

Note: do not attempt to evaluate the integral.

4. Use cylindrical coordinates to compute the volume of the solid region W that lies above the surface $z = x^2 + y^2$ and below the surface $z = 8 - x^2 - y^2$.

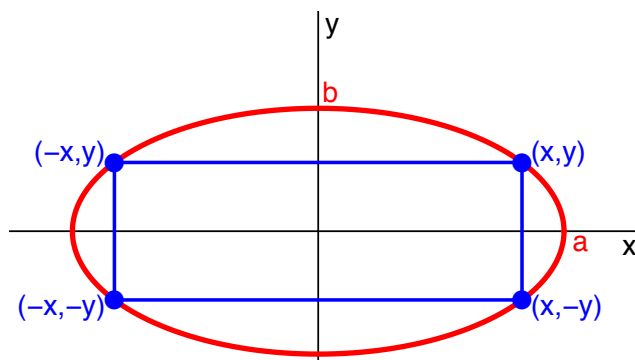
5. The acceleration, initial velocity, and initial position of a particle traveling through space are given by $\vec{a}(t) = (-6, 2, 4)$, $\vec{v}(0) = (2, -5, 1)$, and $\vec{r}(0) = (-3, 6, 2)$, respectively. The particle's trajectory intersects the xz -plane exactly twice. Find these two intersection points.

6. Find an equation for the plane that contains the line $\vec{\lambda}(t) = (-1, 1, 2) + t(3, 2, 4)$ and is perpendicular to the plane $2x + y - 3z = 4$.

7. Evaluate the triple integral $\iiint_W y dV$, where W is the region bounded by

$$z = 0, \quad x + y + z = 4, \quad y - x = 0, \quad x = 2, \quad \text{and} \quad y = 0.$$

8. For each point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, there is a corresponding inscribed rectangle with vertices (x, y) , $(-x, y)$, $(x, -y)$, $(-x, -y)$. See the graphic below. Use the method of Lagrange multipliers to find the inscribed rectangle with maximum area.



9. The two lines $\vec{r}_1(t) = \langle t, 5 - 3t, 5 - 2t \rangle$ and $\vec{r}_2(t) = \langle 3 + 2t, 4 + 2t, 2 - t \rangle$ intersect at a point. Find an equation for the plane that contains both of these lines.
10. The path $\vec{c}(t) = (3 \cos(t), 3 \sin(t), 2t^{3/2})$ traces out a curve $\mathcal{C}$.
- Find the length of the curve $\mathcal{C}$ traced out by $\vec{c}(t)$ for $0 \leq t \leq 3$.
 - Give an example of a path $\vec{r}(t)$ in three dimensions that has constant speed but *nonzero* acceleration. Be sure to show your example has constant speed and nonzero acceleration.