

Math 20C, Fall 2018, Prof. Tesler
Homework #5 Additional Problems
Due Tuesday November 6, 2018

The graded part of the homework is on WebAssign. The problems below are also assigned and you are responsible for doing them, but they will not be collected or graded.

501. Sketch these parametric curves by hand, using methods discussed in class for Chapter 2.4:

- (a) $\vec{c}(t) = (t \cos(t), t \sin(t))$, where $0 \leq t \leq 4\pi$.
- (b) $x = -3t \sin(t)$, $y = 4t \cos(t)$, where $0 \leq t \leq 4\pi$.
- (c) $\vec{c}(t) = -\sin^2(t)\hat{i} + \cos^2(t)\hat{j}$.
- (d) $\vec{c}(t) = 2t\hat{i} + 3\cos(t)\hat{j} + 4\sin(t)\hat{k}$.
- (e) $x = \cos(t)$, $y = \sin(t)$, $z = \sin(3t)$.
- (f) $x = 5e^{-t}$, $y = e^t$, where $0 \leq t \leq \ln(10)$.

After sketching them by hand, check them in Wolfram Alpha (or a graphing calculator):

- Go to <http://www.wolframalpha.com>
 - Convert each formula to the format $(x(t), y(t))$ in 2D or $(x(t), y(t), z(t))$ in 3D. Some other systems instead require the format $x = f(t)$, $y = g(t)$, $z = h(t)$.
 - In the query box, generate one plot at a time using the notation shown below:
Without a range: `parametric plot (3t,cos(t))`
With a range: `parametric plot (3t,cos(t)), t=-2 to 10pi`
 - If you don't specify a range, the default values it uses may or may not be suitable.
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502. Recall the first part of the Fundamental Theorem of Calculus:

$$\text{Let } F(x) = \int_a^x f(t) dt. \text{ Then } F'(x) = f(x).$$

As an example, if $F(x) = \int_a^x \sin(\sqrt{t+8}) dt$, then $F'(x) = \sin(\sqrt{x+8})$.

The integral $\int_0^x e^{-t^2} dt$ arises in probability and statistics. It can't be evaluated in terms of ordinary functions, but it can be evaluated using numerical methods and Taylor series. We'll take derivatives of it.

- (a) Compute $\frac{d}{dx} \int_0^x e^{-t^2} dt$.
- (b) Compute $\frac{d}{dx} \int_0^{x^3} e^{-t^2} dt$ by the following steps:
 - Let $G(u) = \int_0^u e^{-t^2} dt$.
 - Note $\int_0^{x^3} e^{-t^2} dt = G(x^3)$.
 - Use the chain rule to compute $\frac{d}{dx} G(x^3)$.
- (c) Compute $\frac{\partial}{\partial x} \int_{10y}^{x^3} e^{-t^2} dt$ and $\frac{\partial}{\partial y} \int_{10y}^{x^3} e^{-t^2} dt$.
Note $\int_{10y}^{x^3} e^{-t^2} dt = G(x^3) - G(10y)$, using the same $G(u)$ defined in part (b).