Math 188, Winter 2001 February 14, 2001
Prof. Tesler

Homework #6, Due February 21

Chapter 3# 15
Chapter 4# 1, 24
and the problem below: H-3

Problem H-3. Consider the "Change Problem" described on pages 134–8.

(a) Give an example for which the greedy algorithm on page 135 does NOT give the minimum number of coins possible: give the coin denominations; an amount of money, n ¢, to make change for; the quantities of each coin the greedy algorithm says to use for n ¢; and an optimal solution using fewer coins than this. **Do not use the examples given in the book.**

In the remaining questions, the available coins are standard U.S. pennies, nickels, dimes, and quarters:

(b) Write a procedure, modeled on the greedy algorithm on page 135, to make change for n ¢ using pennies, nickels, dimes, and quarters. It should print out how many coins of each denomination to use.

(c) Prove that this algorithm uses the minimum number of coins possible. To get you started, here is the sketch of a proof.

(i) **"Proof of the greedy-choice property":** Prove that there is always an optimal solution that uses the first coin the greedy algorithm says to use, by considering the cases for the first coin. If the first coin the greedy algorithm chooses is a penny, then $1 \leq n \leq 4$ (why?) so the only way to make change is to use n pennies (hence it is true that some optimal solution uses at least one penny). If the first coin is a nickel then $5 \leq n \leq 9$ (why?), and some optimal solution must have at least one nickel (why?). Complete the details, and continue in this way for the other cases.

(ii) **"Proof of optimal substructure":** Prove that to make change for n ¢, if the first coin the greedy algorithm chooses is k ¢, then adding a k ¢ coin to any optimal solution of the $n - k$ ¢ problem gives an optimal solution to the n ¢ problem.

(iii) Use these facts to give a **proof by induction:** This is the kind of induction where you assume that a statement is true for all values from 0 to $n - 1$ and then prove it is true for n.