

Math 186, Winter 2020, Prof. Tesler
Homework #7, Due Wednesday February 26, 2020

Larsen & Marx **Sixth Edition:**

3.2# 22, 24

3.7# 5

4.3# 10, 16, 26, 34

5.3# 2, 5, 6, 7, 10, 14, 20

and the problems below: H-701, H-702.

Problem H-701. During meiosis, two genes on the same chromosome are recombined if an odd number of crossovers occurs in-between them. Let d be the distance in Morgans between two genes.

- (a) Show that the probability that the number of crossovers is odd equals $e^{-d} \sum_{k=0}^{\infty} \frac{d^{2k+1}}{(2k+1)!}$.
- (b) Show that $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} = \frac{e^x - e^{-x}}{2}$ (which is called $\sinh(x)$) by using the Taylor series $e^x = \sum_{m=0}^{\infty} \frac{x^m}{m!}$.
- (c) Combine (a) and (b) to derive a formula considered earlier this quarter: if genes A and B are a distance d apart (in Morgans) then the recombination rate between them is $r = \frac{1}{2}(1 - \exp(-2d))$.
- (d) When d is “small,” the distance d in Morgans is approximately the same as the recombination rate as a fraction. Now we will see why.
- (i) Expand r as a Taylor series in d about $d = 0$ to at least three terms.
- (ii) The first nonzero term in the Taylor series is d . Plot $r_1 = d$ and $r_2 = (1 - \exp(-2d))/2$ as functions of d on the same graph. You may copy it from your graphing calculator or use a printout from a graphing program on a computer.
- (iii) Examine the plots and give approximate values of d where r_1 and r are almost equal.

Problem H-702. Suppose that we are able to count the number of crossovers between two genes A and B occurring in the formation of independent gametes. Over the sample examined, we obtain the following results. Estimate how far apart A and B are in cM.

k	# gametes with k crossovers between A and B
0	30
1	25
2	11
3	3
4	1
> 4	0