

**Math 186, Winter 2020, Prof. Tesler**  
**Homework #6, Due Wednesday February 19, 2020**

Larsen & Marx **Sixth Edition**: (see hints below on problems marked \*)

3.11# 9

4.2# 1, 4, 6, 11, 20, 27

4.3# 2

4.4# 5\*, 6

4.5# 8\*

and the problems below: H-601, H-602

**\*Hints on the indicated problems:**

4.4.5. The formula stated in the question for  $F_X(t)$  holds for real  $t \geq 0$ ; you should also determine the formula for  $F_X(t)$  when  $t < 0$ .

4.5.8. Ignore the hint in the book since it uses a technique we are not covering in class. Instead, use the fact that a negative binomial distribution can be written as a sum of independent geometric distributions with the same  $p$ . So a sum of independent negative binomial distributions with the same  $p$  can be turned into a sum of even more independent geometric distributions with the same  $p$ . That in turn can then be turned into a single new negative binomial distribution with a larger “ $r$ ” parameter.

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**Problem H-601.** We will compute the mean of the geometric distribution. (Note: It’s also possible to compute  $E(X^2)$  and then  $\text{Var}(X) = E(X^2) - (E(X))^2$  by steps similar to these.)

(a) Set  $q = 1 - p$ . Show that

$$E(X) = \sum_{k=1}^{\infty} k \cdot q^{k-1} \cdot p$$

(b) Show that the above summation can be rewritten as follows:

$$E(X) = p \cdot \frac{\partial}{\partial q} \sum_{k=1}^{\infty} q^k$$

(c) Evaluate  $E(X)$  using (b) as follows: evaluate the geometric series; replace the sum in (b) by what it evaluates to; and take the derivative  $\partial/\partial q$ .

(d) Plug in  $q = 1 - p$  and simplify to get the final answer.

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**Problem H-602.** This problem concerns Haldane’s model of crossovers, in which crossovers are a Poisson process with rate  $\lambda = 1 \text{ M}^{-1} = .01 \text{ cM}^{-1}$ . Since this is a rate, the Poisson parameter is  $\lambda d$ .

(a) Two genes are on the same chromosome, 270 cM apart. Make a table of the probabilities of  $k$  crossovers occurring in-between them for  $k = -1, 0, 1, 2, 3, 4, 5, 6$ .

(b) What is the probability of 4 or more crossovers between the two genes in (a)?

(c) What is the expected number of crossovers between the genes in (a), and what’s the standard deviation of that? What is the probability that the number of crossovers occurring will exactly equal the expected number?