

Math 184A, Winter 2019, Prof. Tesler
Homework #7, Due Wednesday February 27, 2019

Chapter 9# 1, 12, 28
and Problems H-701 through H-706 below (see both pages)

Problems H-701 through H-703 are based on the Chapter 18.1 slides. The slides and textbook have notation differences (see the last slide for info); use the notation on the slides.

Problem H-701. In the example in the slides, a “Ferris wheel” is a circular arrangement of colored seats. Rotations (but not reflections) of a given coloring count the same. Parts (a–b) concern Ferris wheels with 6 seats, same as in the slides; you may use relevant formulas from the slides without rederiving them from scratch. But some formulas on the slides do not apply to parts (c–d) since they have 8 seats instead of 6.

- (a) How many colorings are there of Ferris wheels on 6 seats if there are 4 possible colors for each seat: red (R), green (G), blue (B), and white (W)?
- (b) How many colorings are there of a 6-seat Ferris wheel with exactly 2 blue, 2 white, and 2 red seats?
- (c) For the 8-seat Ferris wheel problem, determine (i) the orbit of RWGWRWGW ; (ii) the stabilizer of RWGWRWGW ; (iii) the canonical representative of the orbit, using the rule that we take the lexicographically least element as the canonical one.
- (d) For the 8-seat Ferris wheel problem with k colors, compute $|\text{Fix}(\rho^6)|$.

Problem H-702. Consider the 8-bead necklace problem (symmetries are rotations and reflections).

- (a) Compute (i) the orbit of RGBWRGBW ; (ii) the stabilizer; (iii) the canonical representative of the orbit.
- (b) There are k different bead colors available. Determine $|\text{Fix}(\rho)|$, $|\text{Fix}(\sigma)|$, and $|\text{Fix}(\rho^3\sigma)|$.
- (c) Simplify $\rho^5\sigma^3\rho\sigma^2\rho^2\sigma\rho^7\sigma$ into forms (i) $\sigma\rho^k$ and also $\rho^\ell\sigma$, or (ii) ρ^m , as appropriate, where the exponents k, ℓ, m are *nonnegative* and as small as possible among the valid choices of exponents.

Problem H-703.

- (a) How many ways can 8 squares be colored green and 8 squares be colored white on a 4×4 board of 16 squares?
- (b) Repeat (a) assuming that rotations of the board are regarded as equivalent.
- (c) Repeat (a) assuming that rotations and reflections of the board are regarded as equivalent.

Homework problems continue on next page.

Problems H-704 through H-706 are based on Chapter 9.

Problem H-704. Define a directed graph on $1, 2, \dots, 6$, in which $i \rightarrow j$ is an edge iff j/i is a prime number. *Note that 1 is NOT a prime number.*

- (a) Write the graph in the form $G = (V, E)$ where V is the set of vertices and E is the set of edges.
- (b) Draw the graph.
- (c) Write the adjacency matrix.
- (d) Compute the indegree and outdegree of each vertex.

Problem H-705.

- (a) How many simple graphs are there on vertex set $[6]$?
- (b) How many graphs are there on vertex set $[6]$ if we allow loops and allow up to three edges connecting each pair of vertices (and also up to three loops on each vertex)?
- (c) How many simple directed graphs are there on vertex set $[6]$?
- (d) How many simple graphs (undirected) on vertex set $[n]$ have exactly k edges?

Problem H-706. A graph $G = (V, E)$ is *bipartite* if V can be partitioned $V = A \cup B$ (with $A \cap B = \emptyset$) and every edge has one endpoint in A and the other in B . There's an example in the Chapter 9 class slides.

- (a) Determine the formula for the number of simple bipartite graphs on two given sets of vertices, A and B , with sizes $|A| = n$ and $|B| = m$.

Note that a bipartite graph can have multiple edges between two vertices, but cannot have loops. A simple graph cannot have multiple edges and also cannot have loops.

- (b) For the same setup as (a), determine the formula for the number of simple bipartite graphs with exactly k edges.