

Math 184A, Winter 2019, Prof. Tesler
Homework #4, Due Wednesday February 6, 2019

Note: If you would like to use your homework to study for the exam, please photocopy it before turning it in.

Chapter 6# 1, 2, 5, 30

Chapter 7# 3, 4, 7, 19, 25, 26

and the problems below, H-401 through H-406.

Problem H-401. (Chapter 6) In each part, a permutation is given in one-line, two-line, or cycle form. Convert it to the other two forms. Give its inverse in all three forms.

- (a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 7 & 6 & 2 & 3 & 4 \end{pmatrix}$ (b) 41827653 (c) $(1, 6, 5)(2)(3, 8, 4, 9)(7)$.

Problem H-402. (Chapter 6) Let $f = (1, 3, 9, 2, 5, 8)(4, 7)(6)$ and $g = (1, 6, 5)(2)(3, 8, 4, 9)(7)$.

Please use “right-to-left composition” to multiply permutations throughout this homework.

- (a) Compute fg using right-to-left composition. Give the result in both cycle form and one-line form.
(b) Compute each of $f^2, f^3, f^4, f^5, f^6, f^7$ in cycle form.
(c) The *order* of a permutation f is the smallest positive integer k for which f^k is the identity permutation $1234 \cdots = (1)(2)(3) \cdots$. For example, the order of $(1, 3, 2)(4, 5, 6)(7)$ is 3, and the order of the identity is 1. Determine the order of f and also the order of g .

Hint: The order can be computed based on the cycle lengths; it is not necessary to actually raise a permutation to every single power until you reach the identity. Study the cycle lengths in (b) and find a pattern involving greatest common divisors or least common multiples.

Problem H-403. (Chapter 6)

- (a) How many permutations of $[11]$ consist of two 3-cycles, one 2-cycle, and three 1-cycles?
(b) Compute $c(5, k)$ (Signless Stirling Numbers of the First Kind) for all k .

Problem H-404. (Chapter 7)

- (a) Compute the number of derangements of size 9. (Don’t list them; there are too many.)
(b) List all derangements of $[5]$ in which $f(1) = 3$ and $f(2) = 5$.
(c) List all permutations of $[5]$ in which the fixed points are 1 and 4 (but no others).
(d) Let $D_n(k)$ be the number of permutations of $[n]$ that have exactly k fixed points. The number of derangements of $[n]$ is the special case $D_n(0) = D_n$. Prove that $D_n(k) = \binom{n}{k} D_{n-k}$.

Problem H-405. (Chapter 7) Consider an ordinary deck of cards with the usual 52 cards. Let n be a positive integer. How many ways can we make an n -card hand containing at least one card from each of the four suits?

Hint: Consider the set of n -card hands that do not contain any hearts. Do the same with other suits.

Problem H-406. (Chapters 5 & 7)

- (a) Compute the Stirling number of the second kind $S(50, 3)$. For these numbers, the formula in Theorem 7.5 is much easier than the recursion in Theorem 5.8.
(b) Compute the number of surjections $f : [50] \rightarrow [3]$.