

Math 184A, Winter 2019, Prof. Tesler
Homework #3, Due Wednesday January 30, 2019

Chapter 5# 8, 12*, 26, 27, 32*

and the problems below, H-301 through H-305 (see both pages).

*Notes:

- Problem 5.12: Your answer should be in terms of the function $p(n)$; you don't need to know an explicit formula for $p(n)$ to do this.
- Problem 5.32: $B(n)$ is the Bell number, defined on page 97; in class, we used notation B_n .

Problem H-301. (Based on Chapter 4.) Let m be a positive integer and k be a nonnegative integer.

(a) Prove that $(-m)_k = (-1)^k(m+k-1)_k$.

Hint: $(-12)_5 = (-12)(-13)(-14)(-15)(-16) = (-1)^5(16)(15)(14)(13)(12) = (-1)^5(16)_5$.

(b) Prove that $\binom{-m}{k} = (-1)^k \binom{m+k-1}{k} = (-1)^k \binom{m+k-1}{m-1}$.

(c) Prove that $\frac{1}{(1+x)^m} = \sum_{k=0}^{\infty} (-1)^k \binom{m+k-1}{m-1} x^k$.

Problem H-302. (This and the remaining problems are based on Chapter 5.)

- (a) Compute the Stirling numbers of the 2nd kind $S(6, k)$ for $0 \leq k \leq 6$, and the Bell number $B(6)$.
- (b) How many set partitions of $[13]$ have type $(4, 2, 2, 1, 1, 1, 1, 1)$?
- (c) Verify equation (5.2) on page 97 for the case $n = 4$. In other words, plug in $n = 4$, compute both sides of the equation as functions of x , and simplify to show they're equal.

Problem H-303.

- (a) Determine $p(5)$ (number of integer partitions of 5).
- (b) Determine $p_k(5)$ (number of integer partitions of 5 into exactly k parts) for all values of k .
- (c) Match up the partitions of 5 that are conjugates. Identify any that are self-conjugate.
- (d) Determine the number of strict integer compositions of 5.
- (e) Determine the number of strict integer compositions of 5 into exactly k parts for all values of k .

Problem H-304.

- (a) How many surjective (onto) functions $f : [6] \rightarrow [4]$ are there?
- (b) How many injective (one-to-one) functions $f : [4] \rightarrow [6]$ are there?

See H-305 on next page.

Problem H-305. A “Simplex” lock has n buttons numbered $1, \dots, n$; see the Chapter 5 slides.

- The combination 35-42-1 means to press 3 and 5 together, then 4 and 2 together, then 1. It uses 5 buttons and has 3 groups of pushes.
- The combination $\emptyset$ means don’t push any buttons; just open it. The combination $\emptyset$ is defined for completeness, but it is not an allowed combination on the actual lock.
- Each group of buttons pushed must consist of at least one button.
- Buttons may not be used more than once, and it is not necessary to use all of the buttons, so the number of buttons in a combination will be between 1 and n (combination $\emptyset$ is not allowed).
- *Note:* Most parts of this problem have a numeric answer, except the 2nd part of (e) has a formula as the answer.

Questions:

- (a) How many combinations are there with a standard 5 button Simplex lock?
- (b) How many combinations are there with a standard 5 button Simplex lock if there are at most 3 groups of buttons pushed?

In (c)–(e), we’ll change the rules. Each part is separate; e.g., don’t combine the new rules on (c) and (d); just do them separately.

- (c) How many combinations are there with a 5 button Simplex lock if *all buttons must be used*? (E.g., 35-2 is an invalid combination since 1 and 4 are not used.)
- (d) How many combinations are there on a 5 button lock with at most 3 pushes, if we only allow one button to be pushed at a time and do not allow buttons to be reused?
- (e) If the rules are changed so that each push has one or more buttons and you can reuse buttons (e.g., 13-235-14 is considered valid), how many combinations would be possible on a 5 button lock using exactly 3 groups of pushes? How about on an n button lock using exactly k groups of pushes?