

Math 154, Winter 2020, Prof. Tesler
Homework #9, Due Friday March 13, 2020

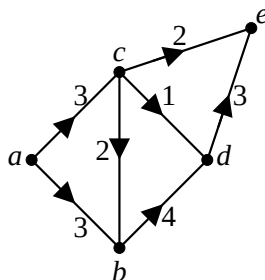
If you want to use this homework assignment to study for the final exam, please make a photocopy of it before you turn it in.

Chapter 8# 8.1*, 8.2*, 8.3*, 8.7*
 and Problems H-901 and H-902 below.

*Hints/clarifications:

- 8.1–8.3: In several of the book's homework problems about flows, the network diagrams just have one number on each edge (the capacity) instead of two (flow/capacity notation), because this is before finding the flow.
- 8.7: Add a source and sink to turn the matching problem into a flow problem; find a maximum flow; and then use that to get a maximum size matching. Note that the concept of augmenting paths for a matching in Chapter 5, and the concept of augmenting paths for a flow in Chapter 8, turn out to have a correspondence.

Problem H-901. A network is shown with source a and sink d . The **capacity** is written on each edge.



- Determine all cuts in the graph, and of those, find which ones have the minimum capacity.
- Find all maximal flows in the graph where the flow on all edges is an integer.
- Is there any maximal flow with real numbers where at least one edge's flow is not an integer?

Problem H-902. Students a, b, c, d, e are applying for internships 1, 2, 3, 4, 5. Students rank the internships from first choice to last choice (listed left to right), and vice-versa:

Student	Preferences	Internship	Preferences
a	3, 2, 1, 4, 5	1	c, e, a, d, b
b	1, 5, 4, 2, 3	2	e, c, a, d, b
c	5, 3, 4, 2, 1	3	a, e, c, d, b
d	4, 1, 5, 2, 3	4	d, b, c, e, a
e	4, 3, 5, 1, 2	5	d, b, c, e, a

- Use the Gale-Shapley algorithm with unmatched students applying to internships at each round, to find a perfect matching.
- Use the Gale-Shapley algorithm with unmatched internships making provisional offers to students each round, to find a perfect matching.
- Consider the matching $M = \{a2, b5, c1, d4, e3\}$. Find an unstable pair (there may be more than one; you only need to find one). That is, student x and internship y are not matched, but they each prefer each other over their current match.