

Math 154, Winter 2020, Prof. Tesler
Homework #8, Due Friday March 6, 2020

Chapter 7# 7.1*, 7.3, 7.5*, 7.7, 7.10, 7.12*
and Problems H-801 and H-802 below.

*Hints/clarifications:

7.1: The four graphs should be called parts (a)–(d) of this problem: (a) (b)
(c) (d)

7.5: We showed that every simple planar graph has a vertex of degree ≤ 5 , and then we used that to show simple planar graphs are 6-colorable. For this problem, the outline of the proof is similar, but it involves different numbers.

7.12: The two “Prove that” statements should be called parts (a) and (b). The degree sequence is from Chapter 1: list the degrees of the vertices in weakly decreasing order. It is not referring to degenerate graphs in Chapter 6, which also involves a sequence of degrees, but defined differently for a different purpose.

Problem H-801.

- (a) A connected graph is drawn in the plane without crossing edges. It has 20 vertices and 45 edges. How many faces does it have?
- (b) An ordinary soccer ball has 32 panels: 20 hexagons and 12 pentagons. (For a picture, see the Wikipedia article “Truncated icosahedron” in the Gallery section.) The vertices, edges, and faces of these panels give a planar graph. An edge and face are *incident* with each other if the edge is contained in the face (just touching at a corner doesn’t count). A vertex and edge or vertex and face are *incident* if the vertex is contained in the edge or face.
 - (i) What degrees do the vertices have? How many faces is each vertex incident with? How many faces is each edge incident with?
 - (ii) Consider the set $T = \{(v, f) : v \text{ is a vertex, } f \text{ is a face, } v \text{ is on } f\}$. Determine its size $|T|$ two ways: first, for each v , consider how many faces it’s in, and add that up over all v ; and second, for each f , consider how many vertices it has, and add that up over all f . Use this to determine the total number of vertices.
 - (iii) Consider the set $U = \{(e, f) : e \text{ is an edge, } f \text{ is a face, } e \text{ is on } f\}$. Determine its size in two ways. Use this to determine the total number of edges.
 - (iv) Show that Euler’s formula (for planar graphs) holds in this graph.

Problem H-802. Prove whether or not this graph is planar. If it is planar, prove it by redrawing it without edges crossing. If it is not planar, use Kuratowski’s Theorem: identify a subgraph homeomorphic to either K_5 or $K_{3,3}$, and draw the graph with that subgraph highlighted and its vertices identified.

