

Math 154, Winter 2020, Prof. Tesler
Homework #6, Due Friday February 21, 2020

Chapter 5# 5.2*, 5.4, 5.6*, 5.7, 5.12*
and Problems H-601 through H-603 below.

*Hints/clarifications:

5.2: The algorithm is to iteratively apply M -augmenting paths to increase the size of the matching, as we did on the class slides.

5.6: A *cubic graph* means 3-regular.

Hint: Let n_1 and n_2 be the number of vertices in the two components. Are these even or odd?

5.12: A 0-1 matrix means all entries are 0's and 1's. The n entries selected as stated in the problem form a *permutation matrix*.

Problem H-601. Complete this 2×5 Latin Rectangle to a 5×5 Latin Square. You only need to give one solution.

1	2	3	4	5
5	3	1	2	4

Problem H-602. Prove that a bipartite graph G must have a matching with at least $\left\lceil \frac{|E(G)|}{\Delta(G)} \right\rceil$ edges. The notation $\lceil x \rceil$ denotes the *ceiling* of x ; if x isn't an integer, then round it up to an integer.

Hint: Use König's Theorem.

Problem H-603.

- (a) If a bipartite graph has a perfect matching, compute $\alpha(G)$, $\beta(G)$, $\alpha'(G)$, and $\beta'(G)$ in terms of the number of vertices, n .
- (b) Give an example of a simple graph that isn't bipartite, where the graph has a perfect matching, but at least one of the four formulas found in (a) does not hold. Compute $\alpha(G)$, $\beta(G)$, $\alpha'(G)$, and $\beta'(G)$ for your example.