

Math 154, Winter 2020, Prof. Tesler
Homework #2, Due Friday January 24, 2020

Please download the latest version of the textbook from the class website (January 14, 2020, or later) before doing this assignment, as it affects some problems.

Chapter 1# 1.1

Chapter 2# 2.9, 2.14(a)*, 2.16, 2.19*

and Problems H-201 through H-203 below.

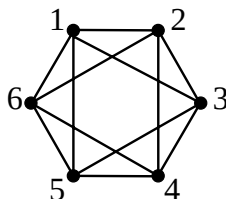
* For #2.14, the two longest paths may or may not be tied for the same length.

For #2.19, see the definition of *orientation* at the beginning of Chap. 1. Also, the graph is finite.

Problem H-201. In each part below, information about a graph is given. Decide if a graph, without loops, exists that satisfies the specified conditions or not. Justify your answers.

- (a) A simple graph with degree sequence $(5, 3, 3, 2, 1, 1)$ (that is, six vertices, where one vertex has degree 5, two vertices have degree 3, one has degree 2, and two have degree 1).
- (b) A multigraph with degree sequence $(5, 3, 3, 2, 2, 1)$.
- (c) A simple graph with degree sequence $(5, 3, 3, 2, 2, 1)$.
- (d) A simple graph with degree sequence $(3, 3, 3, 3)$.
- (e) A tree with six vertices and six edges.
- (f) A disconnected simple graph with 10 vertices, 8 edges, and a cycle.

Problem H-202. These problems refer to this graph:



- (a) Give an example of an Eulerian trail in this graph (starting/ending at different vertices), and also an example of an Eulerian cycle. If either of these do not exist, prove it.
- (b) Give an example of a Hamiltonian path in this graph (starting/ending at different vertices), and also an example of a Hamiltonian cycle. If either of these do not exist, prove it.
- (c) First, give an example of a path of length 4 in the graph from vertex 1 to vertex 2. Second, give an example of a walk of length 4 from vertex 1 to vertex 2, such that it's a walk but is not a path. Recall that the length of a path or walk is the number of *edges* in the sequence of edges.

Problem H-203.

- (a) How many simple graphs are there on four vertices $\{1, 2, 3, 4\}$?
- (b) Find all unlabelled simple graphs on four vertices.
- (c) How many ways can the vertices of each graph in (b) be labelled 1, 2, 3, 4, up to isomorphism?

Hints:

- The graph with four isolated vertices only has one labelling up to isomorphism, not $4! = 24$, because all $4!$ ways to assign the labels to the vertices give the same abstract graph $G = (V, E)$.
- The graph with just one edge has $\binom{4}{2} = 6$ ways to label the vertices of that edge, and the remaining labels are used on the other two vertices, giving a total of 6 ways.
- Your answers to part (c) should add up to the answer of part (a).