

Math 154, Winter 2020, Prof. Tesler
Homework #1, Due Friday January 17, 2020

Please remember, on this and all other homework assignments:

- Present your solutions in the order the problems are listed.
- Show your work on all problems.
- When applicable, circle your final answer.
- On proof-oriented problems, you need to write a coherent proof.
- You may study with others currently enrolled in the class, but the work you turn in should be your own; do not just copy someone else's answer. That includes that you should not copy from an answer manual, answer key, or the internet.

Chapter 1# 1.2, 1.3, 1.6, 1.9*
and Problems H-101 through H-103 below

**Note:* In Question 1.9, the notation $K_{n:r}$ for the Kneser graph, is different than the notation $K_{n,r}$ for a complete bipartite graph.

Problem H-101. Define a directed graph on $1, 2, \dots, 6$, in which $i \rightarrow j$ is an edge iff j/i is a prime number. *Note that 1 is NOT a prime number.*

- (a) Write the graph in the form $G = (V, E)$ where V is the set of vertices and E is the set of edges.
- (b) Draw the graph.
- (c) Write the adjacency matrix.
- (d) Compute the indegree and outdegree of each vertex.
- (e) Compute $N^+(2)$ and $N^-(2)$.

Problem H-102.

- (a) How many simple graphs are there on vertex set $\{1, \dots, 6\}$?
- (b) How many graphs are there on vertex set $\{1, \dots, 6\}$ if we allow loops and allow up to three edges connecting each pair of vertices (and also up to three loops on each vertex)?
- (c) How many simple directed graphs are there on vertex set $\{1, \dots, 6\}$?
- (d) How many simple graphs (undirected) on vertex set $\{1, \dots, n\}$ have exactly k edges?

Problem H-103. A *simple bipartite graph* is a simple graph that's also bipartite.

- (a) Determine the formula for the number of simple bipartite graphs on two given sets of vertices, A and B , with sizes $|A| = n$ and $|B| = m$.
- (b) For the same setup as (a), determine the formula for the number of simple bipartite graphs with exactly k edges.